Productivity Risk and the Dynamics of Stock and Bond Returns

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Abstract

The comovement between returns to stocks and nominal Treasury bonds varies over time in both magnitude and direction. Earlier research attempts to interpret this phenomenon as a consequence of variations in the link between inflation and future economic activity. I present some opposing empirical evidence, and instead argue that in the data, the comovement between stock and nominal bond returns could be driven by real factors. I build a New Keynesian model that generates this behavior through the joint dynamics of output, inflation, and interest rates. The model features two types of persistent shocks to productivity growth: mean-reverting "cyclical" and permanent "trend" shocks. The relative importance of these two shocks varies stochastically over time. I develop a state-space representation of this nonlinear model with stochastic volatility and estimate it using a particle Markov-Chain Monte-Carlo (MCMC) approach. The model could explain the observed patterns in stock-bond return comovement.

Keywords: Production-based asset pricing, Stock returns, Bond returns, Time-varying volatility, New Keynesian model, Nonlinear state-space model, Particle MCMC
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1 Introduction

The stock-bond return correlation is strongly time-varying. In particular, the sign of the correlation turned from positive to negative in the late 1990s. There is a growing literature documenting this time variation using sophisticated statistical models (see, e.g., Guidolin and Timmermann (2006)) but much less work attempting to disentangle its macroeconomic sources. These stylized facts raise the question of what macroeconomic forces determine the risk exposure of U.S. Treasury bonds, and in particular the time variation of risk.

Most papers in the literature such as David and Veronesi (2013), Campbell et al. (2014), Li (2011), Hasseltoft (2009), and Song (2017) focus on the correlation between stock and nominal bonds returns and attempt to explain this phenomenon through variations of the link between inflation and economic activity. This approach appears to be inconsistent with the empirical evidence reported in this paper.

I document novel empirical evidence that the correlation between stock returns and nominal bond returns is closely related to that between stock returns and real bond returns. By using data from both US and UK, I find that this changing pattern of correlation between stocks and bonds applies to both nominal and real bonds. During the mid 1980s, the stock-bond correlation was as high as 60 percent and by early 2000s it dropped to levels as low as -60 percent. What is more striking is that the correlation between stock returns and nominal bond returns move closely with the correlation between stock and real bond returns. This suggests that the correlation between stock and bond return could be driven by real factors.

This paper investigates the possibility that productivity risk is an important source of risk to explain joint movements of stocks and bonds. The contribution of this paper is to add a time-varying real component to a New Keynesian model and show it can jointly account for the dynamics of output, interest rates, inflation, and importantly stock-bond return correlation.

The key mechanism of the model works through the cyclical and trend component of productivity growth. The cyclical component of productivity growth mean-reverts: a positive shock to productivity corresponds to lower expected consumption growth. Lower expected consumption growth translates into lower real interest rates and higher prices for bonds.

Stock and bond returns are, therefore, positively correlated in response to cyclical shocks. The trend component of productivity growth contains a unit root. A positive shock to productivity corresponds to higher expected future productivity growth. Higher expected future productivity growth translates into higher real interest rates and lower prices for bonds. In a New Keynesian framework with recursive preferences, the sign of the correlation between stock returns and bond returns depend on the source of risk. Time-varying relative variance of the cyclical and trend shocks to productivity growth determines the conditional correlation between stock returns and bond returns.

Calibrations and simulations results support the possibility of productivity risk in driving the dynamics of stocks and bonds returns. And the changing magnitude and composition of cyclical and trend shocks perform well in explaining the conditional correlation between stock and bond returns. The model is calibrated to the volatility of cyclical and trend volatility of productivity growth over two samples of US data: pre-1998 and post-1998. Point estimates suggest that the volatility of cyclical productivity shock decreases by around 20 percent from the earlier period to the latter period, while the volatility of trend productivity doubles. The calibrated model approximately matches the stock-bond correlation in both samples .

Subsequently, I investigate the fit of this model with stochastic volatility without imposing breaks in the sample. I develop a state-space representation of this nonlinear model and use a Bayesian particle Monte-Carlo Markov Chain method to estimate it. Due to the nonlinearity nature of the model, the particle filter is used to approximate the likelihood.

The estimation of the model delivers three important empirical findings. First, the model supports the notion that cyclical and trend components are very persistent. The estimated persistence parameter is about 0.95 for the cyclical component, and 0.95 for the trend component. Second, there could be substantial variations in the volatility of cyclical and trend shocks. And using stock and bond returns data in the estimation is necessary to keep track of the volatility movements. Third, despite that the stock-bond return correlation is not directly targeted in the estimation, the estimated model could match the decline in the correlation between stock and bond returns.

2 Some Descriptive Measures of Stock-Bond Return Comovement

This section summarizes some well-known, and some not so well-known, properties of stock and bond returns. Section 2.1 and Section 2.2 focus on U.S. and U.K. markets respectively.

2.1 U.S. Stock-Bond Return Correlation

Figure 1 displays yearly estimates of correlations between aggregate stock returns and returns to both nominal and inflation-indexed long-term Treasury bonds. Yearly estimates of correlation are produced using daily returns. Nominal returns are for the 10-year Treasury bond and real returns are for the 10-year Treasury inflation protected bonds (TIPS). The highest correlation between returns to stocks and returns to nominal bonds is 0.61 in year 1994, and the lowest correlation is -0.63 in year 2012. Guidolin and Timmermann (2006), Baele et al. (2010), Campbell et al. (2014) and other authors all highlight this striking pattern for nominal bonds shown in Figure 1 but don't examine correlation between returns to stocks and returns to real bonds. Returns to stocks and nominal bonds were positively correlated throughout the 1970s, 1980s, and the first half of the 1990s. Estimates of the correlation fluctuated over this period, but on average remain largely positive. In the latter half of the 1990s, estimated correlations dropped sharply to less than zero. Estimates have largely remained negative since then. The pattern carries over to returns calculated using longer holding periods. For example, Figure 2 displays estimates of correlations produced using monthly returns. The estimate for month t is the sample correlation of the 25 returns for months $t-12$ through $t+12$. Although details differ, the message in this figure matches that in Figure 1.

Researchers attempting to explain this large, persistent variation in the stock-nominal bond return correlation largely focus on the changing behavior of monetary policy and/or inflation over the sample. Campbell et al. (2014) argues that changing correlations are driven by regime shifts in the monetary policy reaction function. When the Fed tightens aggressively in response to unexpected increases of inflation, the stock-bond return correlation is more positive. In regimes when the Fed is more accommodating, the stock-bond correlation is more positive. Hasseltoft (2009) studies the implication of changing inflation volatility for stock-bond return correlation. Inflation is assumed to be negatively associated with consumption growth. David and Veronesi (2013) studies the joint dynamics of stock and bonds in an endowment economy with exogenous economic regimes, in which inflation could be either positively or negatively correlated with output growth.

However, evidence in Figures 1 and 2 casts considerable doubt on these stories. Returns to inflation indexed bonds are available beginning with their introduction by the Treasury in 1998. To my knowledge, this is the first paper that emphasizes the correlation between returns to stocks and returns to inflation indexed bonds. A striking result is that during this period estimated correlations of returns to stocks and returns to real bonds closely tracked the stock-nominal bond return correlations. The correlation between these two yearly series (i.e., the correlation between the two time series of yearly estimates of correlations) is 0.71. This tight link suggests that the fundamental determinants of time-varying correlation apply to both real and nominal bonds. It is of course possible that in the mid-1990s there was a large regime change associated with inflation, which cannot be detected using more recent data. We need a longer sample to examine this possibility.

2.2 U.K. Stock-Bond Return Correlation

In the United Kingdom, the history of real bonds goes back to 1986 when inflation was still relatively high. Figure 3 is the U.K. version of Figure 1, displaying yearly estimates of correlations between aggregate stock returns and returns to both nominal and real bonds using daily returns. Stock and nominal bond return correlations are examined by Gusset and Zimmermann (2015), but they do not extend their analysis to real bonds. Nominal returns are for the 10-year gilts and real returns are for the returns the 10-year inflation indexed gilts. [1] The highest correlation between returns to stocks and returns to nominal bonds is 0.59 in 1994, while the lowest correlation observed is -0.61 in 2011. Similar to the finding for U.S., returns to stocks and nominal bonds were largely positively correlated until

[1] Gilts are bonds that are issued by the British government, which are UK equivalent of US Treasury securities. The data is available at http://www.bankofengland.co.uk/statistics/Pages/yieldcurve/archive.aspx

the late 1990s, then largely negative. More importantly, the correlation between returns to nominal bonds and stocks is closely related to the correlation between returns to real bonds and stocks. The correlation between these two yearly series (i.e., the correlation between two time series of yearly estimates of correlations) is 0.97. The correlations between stock returns and inflation-indexed bond returns were also largely positive until the late 1990s, then turned negative. The pattern also applies to returns calculated using longer holding periods. For example, Figure 4 is the U.K. version of Figure 2. Estimates of correlations are produced from monthly returns. The message in this figure largely matches that in Figure 3, which are produced using daily returns. This tight link between returns of nominal and real bonds is consistent with Duffee (2016), which finds that variances of news about expected inflation account for between 10 to 20 percent of variances of yield shocks at a quarterly frequency.

3 The Model

How important are cyclical and trend fluctuations for macroeconomic quantities and prices? To answer this question, I develop a general equilibrium framework to quantitatively account for both macroeconomic and financial moments.

It builds on the standard New Keynesian framework of Woodford (2003) and Galí (2009). There are three standard New Keynesian ingredients. First, the model features imperfect competition in the good market: each firm produces a differentiated good for which it sets the price, given a demand constraint. Second, Calvo (1983) type of price stickiness is introduced by assuming that only a fraction of firms can reset their prices in any given period. Third, the central bank in this economy sets the nominal interest rate according to a Taylor (1993) type rule.

Following the finance literature, households in the economy derive felicity from consumption and leisure following an Epstein and Zin (1989) and Weil (1989) type of utility function. By introducing Epstein-Zin preferences, the model separates the elasticity of intertemporal substitution and risk aversion coefficient and therefore better matches the asset pricing moments.

3.1 Firms

There exists a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good, but they all use an identical technology, represented by the production function

$$Y_t(i) = e^{z_t} K_t^{\alpha} (e^{\Gamma_t} N_t(i))^{1-\alpha} \tag{1}$$

where K_t is the capital stock. The aggregate "final" output is produced from individual goods such that

$$Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

where ϵ measures the degree of substitutability between individual goods.

There are two general approaches in the literature to model shocks to productivity growth. One assumes that productivity growth follows a stationary process; thus the effects of shocks on productivity growth die out over time. This approach is seen in Rudebusch and Swanson (2012) and Kung (2015). Another assumes that productivity growth follows a unit root process, as seen in Croce (2014) and Hsu et al. (2016).

The production function (1) includes both kinds of shocks, which are common across firms. Their relative importance determines the sign of the stock-bond return correlation. The stationary process is z_t, with dynamics

$$z_t = \rho_z z_{t-1} + e^{\sigma_{z,t-1}} \epsilon_{z,t} \tag{3}$$

where $\epsilon_{z,t}$ represents independently and identically distributed draws from a normal distribution with zero mean and standard deviation of 1. Stationarity is imposed by $|\rho_z| < 1$.

The unit root process is Γ_t, with dynamics

$$\Gamma_{t+1} = \Gamma_t + g_t = \sum_{s=0}^{t} g_s$$
$$g_t = (1 - \rho_g)\mu_g + \rho_g g_{t-1} + e^{\sigma_{g,t-1}} \epsilon_{g,t} \tag{4}$$

where $|\rho_g| < 1$, and $\epsilon_{g,t}$ represents independently and identically distributed draws from a normal distribution with zero mean and standard deviation 1. The term μ_g captures the long-run mean growth rate of technology.

The volatility of cyclical and trend shocks follows

$$\sigma_{z,t} = (1-\rho_{\sigma z})\sigma_z + \rho_{\sigma z}\sigma_{z,t-1} + \eta_{\sigma z}\epsilon_{\sigma z,t} \tag{5}$$

$$\sigma_{g,t} = (1-\rho_{\sigma g})\sigma_g + \rho_{\sigma g}\sigma_{g,t-1} + \eta_{\sigma g}\epsilon_{\sigma g,t} \tag{6}$$

The main feature of the process is that the log standard deviations $\sigma_{z,t}$ and $\sigma_{g,t}$ are not constants over time, as commonly assumed. The variation of $\sigma_{z,t}$ and $\sigma_{g,t}$ captures the stochastic volatility of cyclical and trend shocks respectively. The shocks $\epsilon_{\sigma z,t}$ and $\epsilon_{\sigma g,t}$ are normally distributed with mean zero and unit variance. The parameters σ_z (σ_g) and η_z (η_g) controls mean volatility and the standard deviation of shocks to volatility for the cyclical (trend) productivity volatility process. A high σ_z (σ_g) implies a high mean volatility of cyclical (trend) productivity process, and a high $\eta_{\sigma z}$ ($\eta_{\sigma g}$) implies large shocks to cyclical (trend) volatility. Croce (2014) studies a production economy with stochastic volatility where productivity growth follows a unit root.

3.2 Households

We assume that there exists a representative household with Epstein and Zin (1989) and Weil (1989) preferences over the consumption good C_t and leisure L_t with the utility function V_t satisfying:

$$V_t = \left\{(1-\beta)\lambda_t U(C_t, N_t) + \beta\mathbb{E}_t[V_{t+1}^{1-\gamma}]^{\frac{1-\psi}{1-\gamma}}\right\}^{\frac{1}{1-\psi}} \tag{7}$$

where γ is the risk aversion coefficient and ψ is the inverse of intertemporal elasticity of substitution. As highlighted in Albuquerque et al. (2016), I also allow for a preference shock, λ_t, to the time rate of preference. The growth rate of the preference shock $x_{\lambda,t}$ follows an AR(1) process with shocks that are independent of all other shocks in the model

$$x_{\lambda,t} = \rho_\lambda x_{\lambda,t-1} + \sigma_{\lambda,t}\eta_{\lambda,t} \tag{8}$$

where $\eta_{\lambda,t} \sim N(0,1)$. And the volatility $\sigma_{\lambda,t}$ has the same form of dynamics as the cyclical and trend shocks

$$\sigma_{\lambda,t} = (1-\rho_{\sigma\lambda})\sigma_\lambda + \rho_{\sigma\lambda}\sigma_{\lambda,t-1} + \eta_{\sigma\lambda}\epsilon_{\sigma\lambda,t} \tag{9}$$

with the independent shocks to the volatility process denoted by $\eta_{\lambda,t} \sim N(0,1)$. C_t is a consumption index given by

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} \tag{10}$$

The instantaneous utility function is given by

$$U(C_t, N_t) = \begin{cases} \frac{C_t^{1-\psi}}{1-\psi} - e^{\Gamma_t(1-\psi)}\frac{N_t^{1+\varphi}}{1+\varphi} & \text{if} \quad \psi \neq 1 \\ \log(C_t) - e^{\Gamma_t(1-\psi)}\frac{N_t^{1+\varphi}}{1+\varphi} & \text{if} \quad \psi = 1 \end{cases} \tag{11}$$

where $\psi \geq 0$ and $\varphi \geq 0$ determine, respectively, the curvature of the utility of consumption and the disutility of labor. The analysis is considerably simplified by two properties of the above utility function: (1) separability, that is $U_{cn,t} = 0$ and (2) the implied constancy of the elasticities for the marginal utility of consumption and for the marginal disutility of labor. The term $\Gamma_t^{1-\psi}$ is introduced to make the utility function consistent with the notion of balanced growth path as seen in Rudebusch and Swanson (2012). The parameter N_t denotes hours of work or employment. Parameter $\beta \in (0,1)$ is the discount factor. The notation $\mathbb{E}_t\{.\}$ denotes the expectational operator, conditional on information at time t.

The key advantage of using Epstein-Zin utility is that it breaks the link between intertemporal elasticity of substitution and the coefficient of relative risk aversion that has long been noted in the literature regarding expected utility see, e.g., Weil (1989). Household risk aversion to uncertain lotteries over V_{t+1} is amplified by the additional parameter γ, a feature which is crucial for allowing us to fit both the asset pricing and macroeconomic facts below. Note, when $\gamma = \psi$, the utility function coincides with the usual CRRA utility function.

3.2.1 The Marginal Rate of Substitution

The marginal rate of substitution (MRS) between neighboring dates in this economy is given by [2]

$$M_{t,t+1} = \beta \frac{U_{C,t+1}}{U_{C,t}} \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\gamma})^{1/1-\gamma}} \right]^{\psi-\gamma} \tag{12}$$

In the case of $\gamma = \psi$, $M_{t,t+1}$ reduces to the usual formula for the marginal rate of substitution when utility depends only on current period consumption. Therefore, my preference specification nests the class of preferences studied by King et al. (1988).

It is useful to consider an asset that pays C_t as its dividend in each period. This asset is a claim to all future consumption streams $C_{t+1}, C_{t+2}, ...$ In the usual analysis of Epstein-Zin preferences, one substitutes the return on an asset that pays consumption as its dividend into the MRS. Denote the ex-dividend price of this asset as $W_{U,t}$. The return for this asset from t to $t+1$ is defined as

$$R_{W,t+1} = \frac{C_{t+1} + W_{U,t+1}}{W_{U,t}} \tag{13}$$

The appendix shows that the stochastic discount factor (12) can be expressed using the return on this asset as

$$M_{t,t+1} = \left(\beta \frac{U_{C,t+1}}{U_{C,t}} \frac{\lambda_{t+1}}{\lambda_t} \right)^{1-\chi} \left(R_{w,t+1}^{-1} \right)^{\chi} \tag{14}$$

The logarithm of the marginal rate of substitution (MRS) is

$$m_{t+1} = (1-\chi)\rho + (1-\chi)x_{\lambda,t} - (1-\chi)\psi\Delta c_{t+1} - \chi r_{c,t+1}$$

[2]Detailed derivation is provided in appendix.

where $1-\chi = \frac{1-\gamma}{1-\psi}$.

The expression for the marginal rate of substitution in terms of an asset return is useful for two reasons. First, expressing the marginal rate of substitution in terms of asset returns will be important in the implementation of the approximation method for the model. Second, it shows how the marginal rate of substitution changes from the usual form by introducing Epstein-Zin preferences. Instead of the standard setup where only consumption matters, the marginal rate of substitution now depends on the realization of the asset returns.

3.2.2 Budget Constraint

The maximization of utility (7) is subject to a sequence of flow budget constraints given by

$$P_t \left[C_t + K_{t+1} - (1-\delta)K_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - e^{\mu_g}\right)^2 K_t \right] + q_t B_t = W_t N_t + D_t + B_{t-1} \tag{15}$$

Capital depreciates at the rate δ, and changes to the capital stock entail a quadratic adjustment cost [3]

$$\frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - e^{\mu_g}\right)^2 K_t$$

in which $t = 0, 1, 2, \ldots$ The parameter P_t is the price of the consumption good, and W_t denotes the nominal wage (per hour or per worker, depending on the interpretation of N_t). The symbol B_t represents the quantity of one-period nominally riskless discount bonds purchased in period t and maturing in period $t+1$. Each bond pays one unit of money at maturity, and its price is Q_t. Nominal dividends are represented by D_t, accruing to households as the owner of firms. In addition to (15), it is assumed that households are subject to solvency constraint that prevent them from engaging in Ponzi-type schemes. The following constraint is assumed

$$\lim_{T\to\infty} \mathbb{E}_t \left\{ M_{t,T} \frac{B_T}{P_T} \right\} \geq 0 \tag{16}$$

[3] This form of adjustment cost is motivated to ensure there is no adjustment costs in the steady state.

for all t, where $M_{t,T} \equiv \beta^{T-t} U_{c,T}/U_{c,t}$ is the stochastic discount factor. We also use S_t to denote the market value of firms' shares.

3.3 Optimal Price Setting

Following the formalism proposed in Calvo (1983), each firm may reset its price only with probability $1-\theta$ in any given period, independent of the time elapsed since it last adjusted its price. Thus, in each period a measure of $1-\theta$ producers reset their prices, while a fraction of θ keep them unchanged. As a result, the average duration of a price is given by $\frac{1}{1-\theta}$. Therefore, θ is the measure of price stickiness.

A firm reoptimizing in period t, will choose P_t^* that maximizes the current market value of the profits generated while that price remains effective.

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left\{ M_{t,t+k}^{\$} (P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right\} \tag{17}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \tag{18}$$

for $k = 0, 1, 2, ...,$, where $M_{t,t+k}^{\$} \equiv \beta^k (U_{c,t+k}/U_{c,t})(P_t/P_{t+k})$ denotes the nominal stochastic discount factor, and $\Psi_t(.)$ is the cost (nominal) function and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t.

3.4 Central Bank

The central bank in the economy sets the nominal interest rate following a Taylor (1993) policy rule

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[r^* + \phi_y \tilde{y}_t + \phi_\pi \tilde{\pi}_t] + \epsilon_t^v \tag{19}$$

where ϵ_v is an independently and identically distributed stochastic monetary policy shock with mean zero and variance $\sigma_{v,t}^2$. The volatility of monetary policy shock follows

$$\sigma_{v,t} = (1 - \rho_{\sigma v})\sigma_v + \rho_{\sigma v}\sigma_{v,t-1} + \eta_{\sigma v}\epsilon_{\sigma v,t} \tag{20}$$

Time-varying monetary policies have been hypothesized by Song (2017) and Campbell et al. (2014) as sources of change for the stock-bond return correlation. In the presence of time-varying monetary policy shocks, it would then be useful to evaluate how the change in stock-bond return correlation can be attributed to time-varying cyclical and trend shocks. The term $\tilde{y}_t$ denotes the deviation of detrended output from its steady state value where $y_t \equiv \log(Y_t) - \Gamma_t$, and $\tilde{\pi}_t$ denotes the deviation of inflation from its steady state. The term σ_t Coefficients ϕ_π and ϕ_y are chosen by the monetary authority, and assumed to be non-negative. r^* is the steady state level of real interest rate.

3.5 Equilibrium

In equilibrium, nominal wage W_t, the price of goods $P_{i,t}$ and consumption sector inflation π_t are set to clear all markets

- Labor market clearing:
- Consumption-good market clearing:

$$C_t + I_t = Y_t \tag{21}$$

- Zero net supply of bonds:

$$i_t = -\mathbb{E}_t[m_{t,t+1}^{\$}] \tag{22}$$

An equilibrium consists of prices and allocations such that (a) taking prices and wage as given, each household's allocations solves (7); (b) taking aggregate prices and wage as given, firm's allocation solve (17) ; (c) labor, consumption-good and bond markets clear. I'm solving for a symmetric equilibrium, in which all intermediate good firms choose the same price P_t, employ the same amount of labor N_t and choose to hold the same amount of capital K_t.

3.6 Equity Pricing

I use a standard approach from the asset pricing literature that the stock market in this model is a leveraged claim on future aggregate consumption. In each period, it pays out consumption units D_t. The log of aggregate dividend is scaled log consumption.

$$d_t = \phi c_t \tag{23}$$

The parameter ϕ is capturing a broad concept of leverage, including operating leverage. The interpretation of dividends as a levered claim on consumption is common in the asset pricing literature (Abel (1990), Campbell (2003), Bansal and Yaron (2004) and etc).

Let $W_{S,t}$ be the real price of stocks, the law of one prices implies that

$$W_{S,t} = \sum_{s=1}^{\infty} \mathbb{E}_t[M_{t,t+s} D_{t+s}] = \mathbb{E}_t[M_{t,t+1}(D_{t+1} + W_{S,t+1})] \tag{24}$$

3.7 Bond Pricing

The Euler equation implies that the price of nominal bonds satisfies that

$$\mathcal{P}^{\$}_{n,t} = \mathbb{E}_t(M_{t,t+1} e^{-\pi_{t+1}} \mathcal{P}^{\$}_{n-1,t+1}) \tag{25}$$

where $\mathcal{P}^{\$}_{n,t}$ is the price of a zero-coupon bond that matures on date $t+n$ and pays 1 dollar at time $t+n$.

The yield-to-maturity on the n -period nominal bond is defined as

$$\mathcal{Y}^{\$}_{n,t} = -\frac{1}{n} \mathcal{P}^{\$}_{n,t} \tag{26}$$

Similarly, the price of a n -period real bond can be written as

$$\mathcal{P}_{n,t} = \mathbb{E}_t[M_{t,t+1}\mathcal{P}_{n-1,t+1}] \tag{27}$$

and the corresponding yield-to-maturity is defined as

$$\mathcal{Y}_{n,t} = -\frac{1}{n}\mathcal{P}_{n,t} \tag{28}$$

4 Quantitative Implications

This section discusses the quantitative implications of the model. As the relative importance of the productivity shocks in (3) and (4) could drive the sign of the stock-bond return correlation, the intuition is easiest to see through a comparative statics exercise by comparing two cases with fixed volatilities but where the relative importance of the cyclical and trend shock differ. Therefore, the model analyzed in this section is the one without stochastic volatility with the following assumption.

Fixed Volatility Assumption:

$$\sigma_{z,t} = \sigma_{z,\text{case}}, \quad \sigma_{g,t} = \sigma_{g,\text{case}} \tag{29}$$

The fixed volatility specification of productivity shocks follows Aguiar and Gopinath (2007). My approach differs from theirs both in the focus (they examine capital flows of emerging markets) and in the choice of parameters.[4] In both sample, the volatility of monetary policy shocks and preference shocks are calibrated to be the same , i.e., $\sigma_{v,t} = \sigma_v, \quad \sigma_{\lambda,t} = \sigma_\lambda$, to focus on the importance of productivity shocks in driving stock-bond return correlation.

4.1 Data and Summary Statistics

I use quarterly US data on output, inflation, interest rates, and aggregate stock returns from 1960Q1-2015Q4. The productivity measure used is the labor productivity measure

[4] Naturally, the fixed volatility assumption is inconsistent with the motivating evidence that correlations change over time. It also oversimplifies the asset-pricing setting, since investors do not have to consider the possibility that relative volatilities will vary.

from the Bureau of Labor Statistics. [5]

4.2 Calibration

Table 1 presents the quarterly calibration for the parameters of the model. In this section, I employ a model that leaves out many of the nominal frictions in standard business cycle work in order to focus on the ability of the particular mechanism just described to generate realistic nominal and real stock-bond return correlation. Panel A reports the values for preference parameters. The elasticity of intertemporal substitution $1/\sigma$ is set to 0.5, which is consistent with estimates in the micro literature (e.g., Vissing-Jorgensen (2002)) [6]. The coefficient of relative risk aversion is set to 10.0, which is standard values in the long-run risk literature (e.g., Bansal and Yaron (2004)). The subjective discount factor is calibrated to be 0.99.

Panel B reports the calibration of technological parameters. The desired markup is set to be 1.2. The capital share α is set to 0.33, and the depreciation rate of capital is set to 0.02. These three parameters are calibrated to standard values in the macroeconomic literature. The price adjustment parameter θ is set to be 0.75, meaning that 25 percent of firms adjust their prices in each period.

Panel C reports the parameter values for the productivity process. The quarterly persistence parameter ρ_z is calibrated to 0.95 to match the first autocorrelation of expected productivity growth. This value is in line with Rudebusch and Swanson (2012) and Kung and Schmid (2010). The quarterly persistence parameter of the trend shock is also set to be 0.98, which is in line with the monthly persistence parameter about 0.99 in Bansal and Yaron (2004) and Schorfheide et al. (2018). In the next section, I estimate these persistence parameters using the model with stochastic volatility, and find that these calibrated values are close to the estimates.

Panel D reports the calibration of the monetary policy rule parameters. The parameter governing the sensitivity of the interest rate to inflation ρ_π is set to 1.5. The parameter determining the sensitivity of the interest rate to output ρ_y is set to 0.1. The persistence

[5] The data is available at http://download.bls.gov/pub/time.series/pr/

[6] I also attempt to estimate the parameter σ using the model with stochastic volatility and find that σ is significantly above 1.

of monetary policy shock ρ_i is set to be 0.5. The volatility of interest rate shocks σ_v is set to 0.3%. The magnitude of the preference shock is set to be 0.04%, which is close to the estimate in Albuquerque et al. (2016). These parameter values are standard in the literature.

4.3 Evaluating the Fit of the Model

The goal of the current exercise is to see in a comparative static sense whether the model reproduces observed stock-bond return correlation. The model is calibrated to two periods of productivity growth differing only in the volatility of cyclical and trend shocks. The model is solved in Dynare using a second-order approximation. I find that it can both provide a reasonable fit to the usual business cycle properties of the data, and importantly produce the striking change in the stock-bond return correlation discussed above. Table 2 summarizes the model fit for two subperiods of US economy: pre-1998 and post-1998.

4.3.1 Estimating the Volatility of Cyclical and Trend Shocks

The magnitude of cyclical and trend shock volatility for both samples is estimated from productivity data in pre-1998 and post-1998. In each sample, I conduct a maximum-likelihood estimation of the productivity processes for the volatility of cyclical and trend shocks. The estimates of the volatility are directly fed into the model. They are reported in the first two columns in Table 2. The volatility of the cyclical shock is about 12 times larger than the volatility of the trend shock in the pre-1998 sample. In the post-1998 sample, the volatility of cyclical shock is about 10 times larger than that of the trend shock.

4.3.2 Evaluating the General Adequacy to Macroeconomics Moments

Panel A shows that the model fits standard deviations of detrended output growth, inflation, and detrended wage rates moderately well. [7] For example, the standard deviation of detrended output is about 1.62 percent in the pre-1998 sample whereas the model produces 2.21 percent. Panel B shows that the model could also moderately match the first-order autocorrelations of detrended output and inflation.

[7] I use a Hodrick and Prescott (1997) filter with smoothing parameter 1600 to detrend output and wages.

4.3.3 Evaluating the Adequacy to Key Financial Moments

Panel C shows the model approximately matches the key financial moment: stock-bond return correlation. In the pre-1998 sample, the model produces a stock-bond correlation of 0.41 while the correlation the data is 0.37. In the post-1998 period, the model produces a stock-bond correlation of -0.09 while it is -0.25 in the data. Another important feature of data that the model is able to approximately fit is the correlation between changes in yields and changes in the slope of the yield curve. [8] Because cyclical shocks are mean-reverting and short-lived, they have larger effects on short-term interest rates relative to long-term ones. On the contrary, trend shocks are long-lived and therefore have bigger effects on the long-term interest rates. Therefore, the slope of the yield curve decreases in response to a positive cyclical shock whereas it increases in response to a positive trend shock. The correlation between changes in long-term interest rates and changes in the slope of the yield curve is thus negative following cyclical shocks and positive with respect to trend shocks.

Figure 5 plots the 5-year moving correlation between changes of the 5-year zero coupon bond yields and the slope of the yield curve. [9] The correlation is mostly negative in the 1970s and 1980s and becomes positive in the recent decades. This pattern is striking as it is the opposite of the movement we see for stock-bond returns correlation. Nevertheless, this pattern is not implied by the correlation between stock-bond returns. Therefore, it could serve as an important validation for the key mechanism of the model. I evaluate the fit of the model to the correlation between changes in 5-year yields and changes in yield curve slopes. In the pre-1998 period, the model produces a correlation of -0.04, compared with -0.22 in the data. In the post-1998 period, the model produces a correlation of 0.19, compared with 0.49 in the data. The model underpredicts the correlation in the post sample. A potential reason is that the short-term interest rate in the U.S. has been stuck at the zero-bound in recent years, whereas the model has no zero lower bound. Therefore, changes of slopes are strongly positively correlated with changes of long-term yields.

[8] The slope of the yield curve in general is defined as the long-term interest rates minus the short-term interest rates.

[9] The slope is measured by the 5-year yield less the three-month bill rate.

4.4 Impulse Responses

Impulse response functions clarify the mechanisms by which individual shocks act on stocks, bonds, and macroeconomic variables. This section presents the impulses of variables to cyclical and trend productivity shocks.

Figure 6 shows responses of output, inflation, nominal interest rate, the yield for 10-year nominal and real bonds and stock prices to a cyclical productivity shock. A cyclical shock acts as a strongly positive impulse to output, but as a negative one to inflation. It increases output, lowers the marginal production cost and therefore inflation. Long-term real interests fall as people expect the economy to go back to the long-term trend. Both stock and bond prices increase following a cyclical shock, so cyclical productivity shocks tend to raise stock-bond correlation.

Figure 7 shows responses of output, inflation, nominal interest rate, the yield for 10-year nominal and real bonds and stock prices to a a trend shock. A trend shock acts as a strongly positive impulse to nominal and real short-term interest rates. Because it implies a large wealth effect to consumers, households increase consumption. The output and inflation therefore increases as well. Stock prices increase significantly following a trend shock, while bond prices fall. Thus, trend shocks tend to decrease stock-bond return correlation.

5 Bayesian Estimation of the Model with Stochastic Volatility

This section investigates and estimates the model with stochastic volatility. I use a state-space representation that facilitates the estimation. Let z_t denotes the vector of state variables in deviation from the deterministic steady state (excluding volatility states $\sigma_{z,t}$, $\sigma_{g,t}$, $\sigma_{v,t}$ and $\sigma_{p,t}$) and $\sigma_t = [\sigma_{z,t}, \sigma_{g,t}, \sigma_{v,t}, \sigma_{p,t}]$ as the vector of volatility states. The vector of all state variables is defined as $s_t \equiv [z_t, \sigma_t]$. Also, the observable series used to estimate the model are denoted as x_t, which could consists of productivity growth, bond yields, and stock returns. Let $\epsilon_t = [\epsilon_{z,t}, \epsilon_{g,t}, \epsilon_{v,t}, \epsilon_{p,t}]$ denotes the vector of shocks , and $\epsilon_\sigma = [\epsilon_{\sigma z}, \epsilon_{\sigma g}, \epsilon_{\sigma v}, \epsilon_{\sigma p}]$ represents shocks to the conditional volatilities. And $v(.)$ is function that depends on the volatility of the shocks. The approximated solution of the model can be characterized by a state-space representation in the following form

$$z_t = \Phi(\kappa) z_{t-1} + v(\sigma_t)\epsilon_t \tag{30}$$

$$x_t = \mu_x(\kappa) + \mathrm{T}(\kappa) s_{t-1} + \omega_t \tag{31}$$

The parameter κ is the collection of parameters of the model. Equation (30) characterizes the evolution of state variables as a vector autoregressive process with stochastic volatility. The term $v(\sigma_t)$ captures stochastic volatility in productivity growth, which drives the dynamics of state variables. Economic observables x_t are linked with state variables through the measurement equation (31), where ω_t is the measurement error. This approximated solution attempts to simplify the representation of the model, while maintaining the key feature of stochastic volatility. In the calibration section, I find that solving the model in higher order terms doesn't impact the movements of the model tremendously.

The model with stochastic volatility captured by (30) and (31) is a nonlinear state space model whose log-likelihood function is not known in closed form. Thus, I use a particle filter to approximate the likelihood of the model and subsequently embed the likelihood approximation into a random-walk Metropolis–Hastings algorithm. The details of the estimation are provided in the appendix.

5.1 Empirical Analysis

In this section, I estimate the stochastic volatility version of the model using three observables: productivity growth, stock returns, and bond returns. The data spans from 1960:Q1 to 2016:Q4. The bond return is the return on five-year bond from the CRSP US Treasury Database, and the stock return is the return on S&P 500.

There are in total 16 parameters to be estimated. The persistence parameters include ρ_z, ρ_g, ρ_i, and ρ_p. The persistence of volatility processes are captured by $\rho_{\sigma z}$, $\rho_{\sigma g}$, $\rho_{\sigma v}$ and $\rho_{\sigma p}$. Let σ_z, σ_g, σ_v, and σ_p denote the steady state log standard deviation of cyclical shocks, trend shocks, monetary policy shocks and preference shocks, $\eta_{\sigma z}$, $\eta_{\sigma g}$, $\eta_{\sigma v}$, $\eta_{\sigma p}$ denote the standard deviation of shocks to volatility processes.

To constrain the number of parameters used in estimation, I set other parameters the same as those used in the calibration section. The average of the productivity growth rate μ_g is set to be 0.8% such that the model's long-term average of productivity growth $(1-\alpha)*\mu_g$ matches the historical average of productivity growth.

I now proceed to the Bayesian estimation of the model. Table 3 summarizes posterior distributions of estimated parameters and their priors in the estimation. The prior for these parameters are relatively flat. For the persistence parameters ρ_z, ρ_g, ρ_v, ρ_p, $\rho_{\sigma,z}$, $\rho_{\sigma,g}$, $\rho_{\sigma,v}$, and $\rho_{\sigma,p}$, I choose a uniform prior between -1 and 1. The prior distribution for the log volatility of shocks is normally distributed with a mean of -5 and standard deviation of 2. Standard deviations of shocks to the volatilities are set to be inverse gamma distributed with mean 0.2 and standard deviation of 0.2.

The posterior medians of the persistent parameters ρ_z, ρ_g, ρ_i, and ρ_p are 0.87,0.98, 0.51, and 0.95 respectively. These persistence parameters at the quarterly frequency suggest that the cyclical and trend components are very persistent. The monetary policy shock is relatively short-lived, but the preference shock is persistent. And the confidence intervals for persistence parameters are small. These values also give support to the choice of parameters used in the calibration section. As for the persistence parameters for the volatility, the posterior median of $\rho_{\sigma,z}$ and $\rho_{\sigma,g}$, $\rho_{\sigma,v}$, $\rho_{\sigma,p}$ are 0.87, 0.91, 0.96, 0.81 respectively. So the volatilities of these shocks are persistent.

5.2 The Fit of Stock-Bond Return Correlation

Despite that the stock-bond return correlation is not a directed targeted moment in the estimation, I evaluate the fit of the model in explaining the observed stock and bond return correlation. The stock-bond return correlation changed from positive to negative around 1998. Figure 8 plots the smoothed volatility of the cyclical shocks, trend shocks, monetary policy shocks, and preference shocks. We can see that there is a decline in the volatility of cyclical shocks. And the volatility of trend shocks increases around year 1998. This result is consistent with the split-sample calibration using productivity series alone, which lends support to the notion that productivity risk is an important force in driving stock and bond returns [10]. And the use of stock and bond returns data allows us the keep track of these volatilities with small confidence intervals.

Figure 9 plots the model-implied stock-bond return correlation. We can see that the model is successful in capturing a decline of the stock-bond return correlation. The pattern is broadly consistent with the realized stock-bond return correlation in Figure 1. The estimated model doesn't fully match the level of the stock-bond correlation, though.

6 Conclusion

This paper has examined the importance of productivity shock in driving the dynamics of stock and bond return through a New Keynesian model with two types of persistent shocks to productivity growth. The model features two types of persistent shocks to productivity growth: mean-reverting "cyclical" shocks and permanent "trend" shocks. The relative importance of these two shocks varies stochastically over time.

I develop a state-space representation of this nonlinear model with stochastic volatility and estimate it using a particle Markov-Chain Monte-Carlo (MCMC) approach. Empirical analysis finds that cyclical and trend shocks to productivity can be important source to account for asset prices and macroeconomics quantities. The cyclical fluctuations of productivity growth lead to large positive comovement between stock and bond returns, while the trend fluctuations give rise to large negative comovement between stock and bond returns.

The main mechanism identified by the model provides several additional testable implications, such as the covariance between short and long term interest rates. The cyclical fluctuations are associated with more movement in short-term interest rates relative to that in the long-term rates. The trend fluctuations are associated with more movements in long-term interest rates. This pattern is broadly consistent with the data.

[10] The stochastic volatility model assumes that volatilities are stationary over time. Therefore, it may underpredict the potential regime change in the magnitude of the volatilites.

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Tables and Figures

Table 1: Quarterly Calibration

Parameter	Description	Value
Panel A: Preferences		
β	Subjective discount factor	0.99
ψ	Inverse of elasticity of intertemporal substitution	2
γ	Risk aversion	10.0
φ	Inverse of labor supply elasticity	0.3
Panel B: Technology		
α	Capital share	0.33
δ	Depreciation rate of capital stock	0.02
θ	Price adjustment frequencies	0.75
Panel C: Productivities		
ρ_z	Persistence of z	0.95
ρ_g	Persistence of g	0.98
Panel D: Monetary Policy		
ρ_i	Degree of monetary policy inertia	0.5
ϕ_y	Sensitivity of interest rate to output	0.1
ϕ_π	Sensitivity of interest rate of inflation	1.5

This table reports the parameter values used in the quarterly calibration of the model. The table is divided into four categories: preferences, technology, firms price setting and policy parameters.

Table 2: Calibrated Model Moments

Sample Period	1960Q1-1998Q4		1999Q1- 2015Q4	
Statistic	Data	Model	Data	Model
Volatility Parameters				
σ_z	0.84	0.84	0.66	0.66
σ_g	0.031	0.031	0.068	0.068
Panel A: Standard deviations				
$\sigma(y)$	1.62	2.21	1.20	2.09
$\sigma(w)$	1.11	3.85	1.56	3.87
Panel B: Autocorrelations				
$AC_1(y)$	0.98	0.91	0.94	0.87
$AC_1(\pi)$	0.88	0.96	0.50	0.97
Panel C: Correlations				
$corr(\Delta\mathcal{Y}_5^\$ - \Delta\mathcal{Y}_{3m}^\$, \Delta\mathcal{Y}_5^\$)$	-0.22	-0.04	0.49	0.19
$corr(\pi, \Delta c)$	-0.32	-0.05	0.11	0.01
$corr(r_w, r_{10}^\$)$	0.37	0.41	-0.25	-0.09

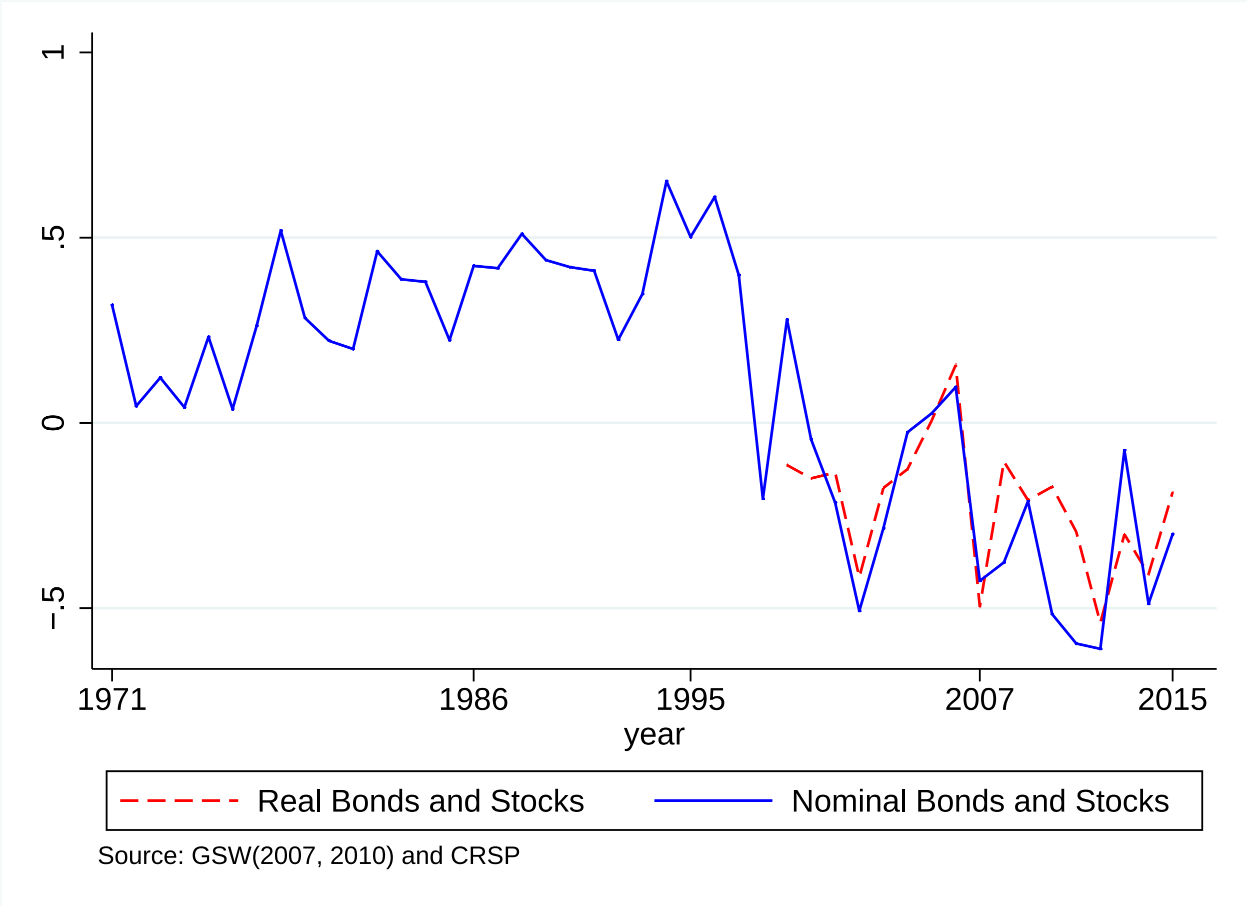
This table presents the standard deviations, autocorrelations, and cross-correlations for key economic variables from the data and from the model. The model is calibrated at a quarterly frequency and the reported statistics are annualized.

Table 3: Prior and posterior distributions of estimated parameters

Parameter	Distribution	Prior			Posterior		
		5%	50%	95%	5%.	50%	95%
Cyclical Shock							
ρ_z	U	-0.90	0	0.90	0.84	0.87	0.90
$\rho_{\sigma z}$	U	-0.90	0	0.90	0.85	0.87	0.90
σ_z	N	-9	-5	-1	-4.81	-4.77	-4.75
$\eta_{\sigma z}$	IG	0.06	0.15	0.49	0.09	0.10	0.11
Trend Shock							
ρ_g	U	-0.90	0	0.90	0.95	0.98	0.99
$\rho_{\sigma g}$	U	-0.90	0	0.90	0.90	0.91	0.92
σ_g	N	-9	-5	-1	-7.05	-6.95	-6.91
$\eta_{\sigma g}$	IG	0.06	0.15	0.49	0.16	0.18	0.20
Monetary Policy Shock							
ρ_i	U	-0.90	0	0.90	0.49	0.51	0.52
$\rho_{\sigma v}$	U	-0.90	0	0.90	0.95	0.96	0.99
σ_v	N	-9	-5	-1	-5.76	-5.71	-5.66
$\eta_{\sigma v}$	IG	0.06	0.15	0.49	0.03	0.03	0.05
Preference Shock							
ρ_p	U	-0.90	0	0.90	0.98	0.99	0.99
$\rho_{\sigma p}$	U	-0.90	0	0.90	0.77	0.81	0.85
σ_p	N	-9	-5	-1	-6.81	-6.75	-6.53
$\eta_{\sigma p}$	IG	0.06	0.15	0.49	0.07	0.09	0.10

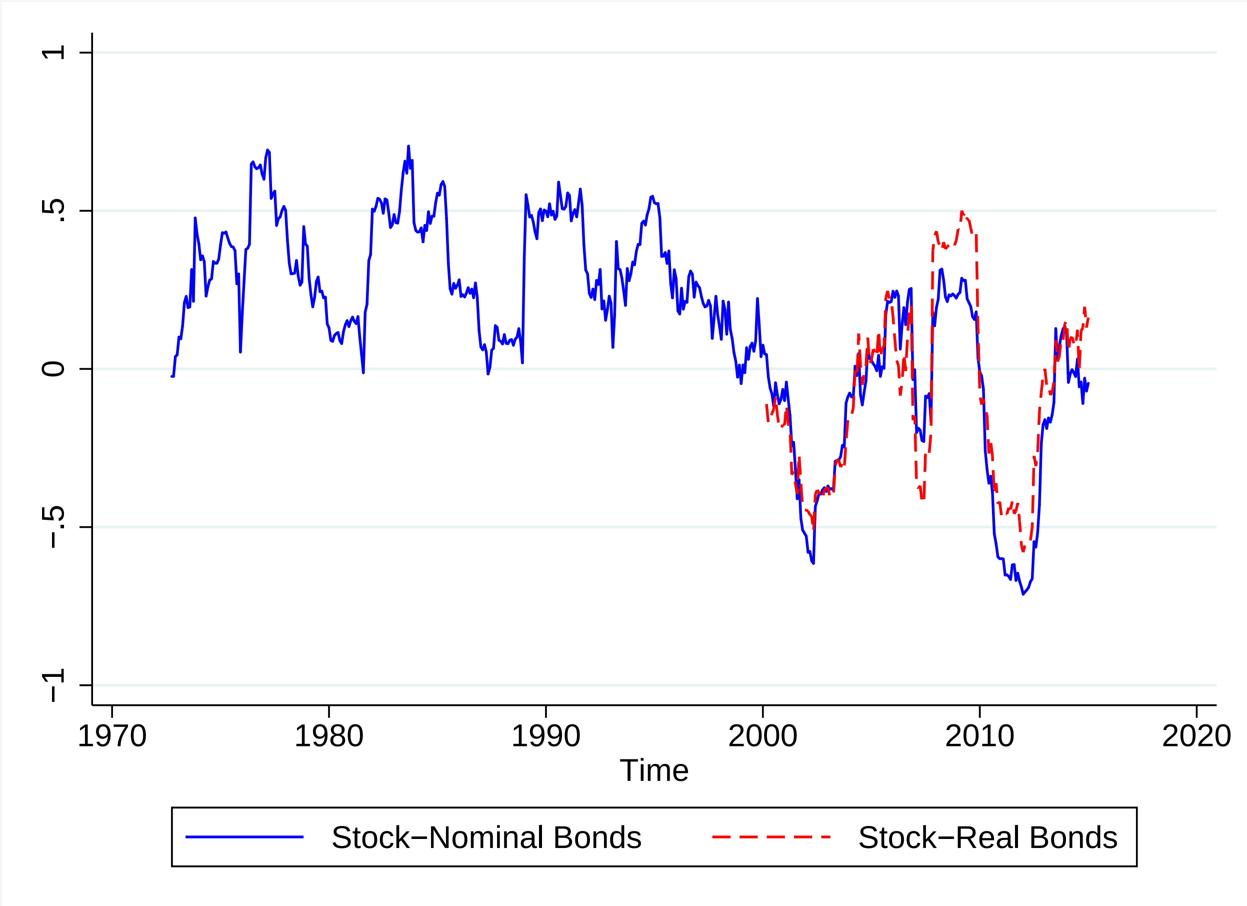
This table reports the prior and posterior distribution of parameters from the estimation of the model. There are 16 parameters estimated. U, N, and IG denote normal, uniform and inverse gamma distribution respectively.

Figure 1: Realized Stock-Bond Correlation for U.S.



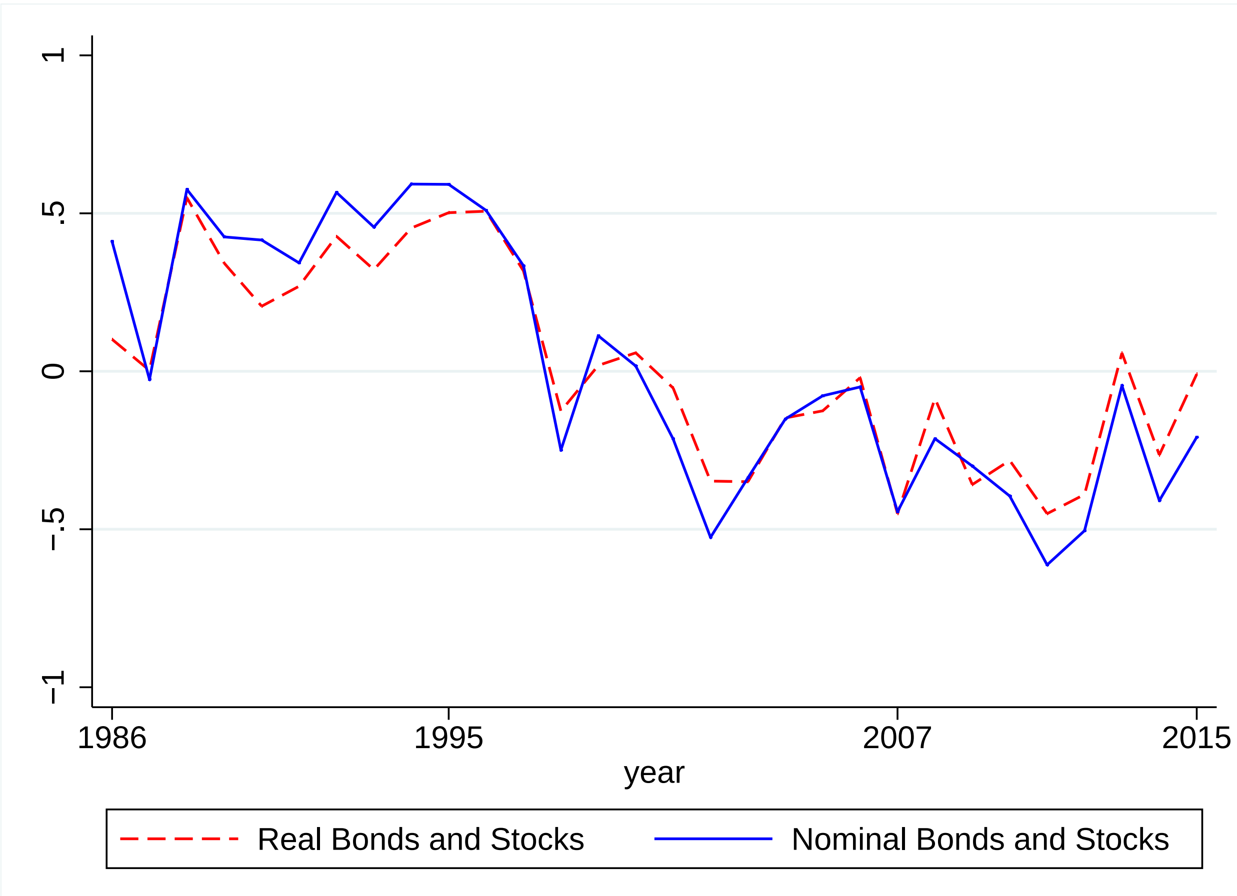
This figure graphs realized quarterly correlations measured using daily returns for nominal and real bonds in U.S. The data used for real bonds, which are known as the TIPs, starts at 1998.

Figure 2: Moving Stock-Bond Correlation for U.S.



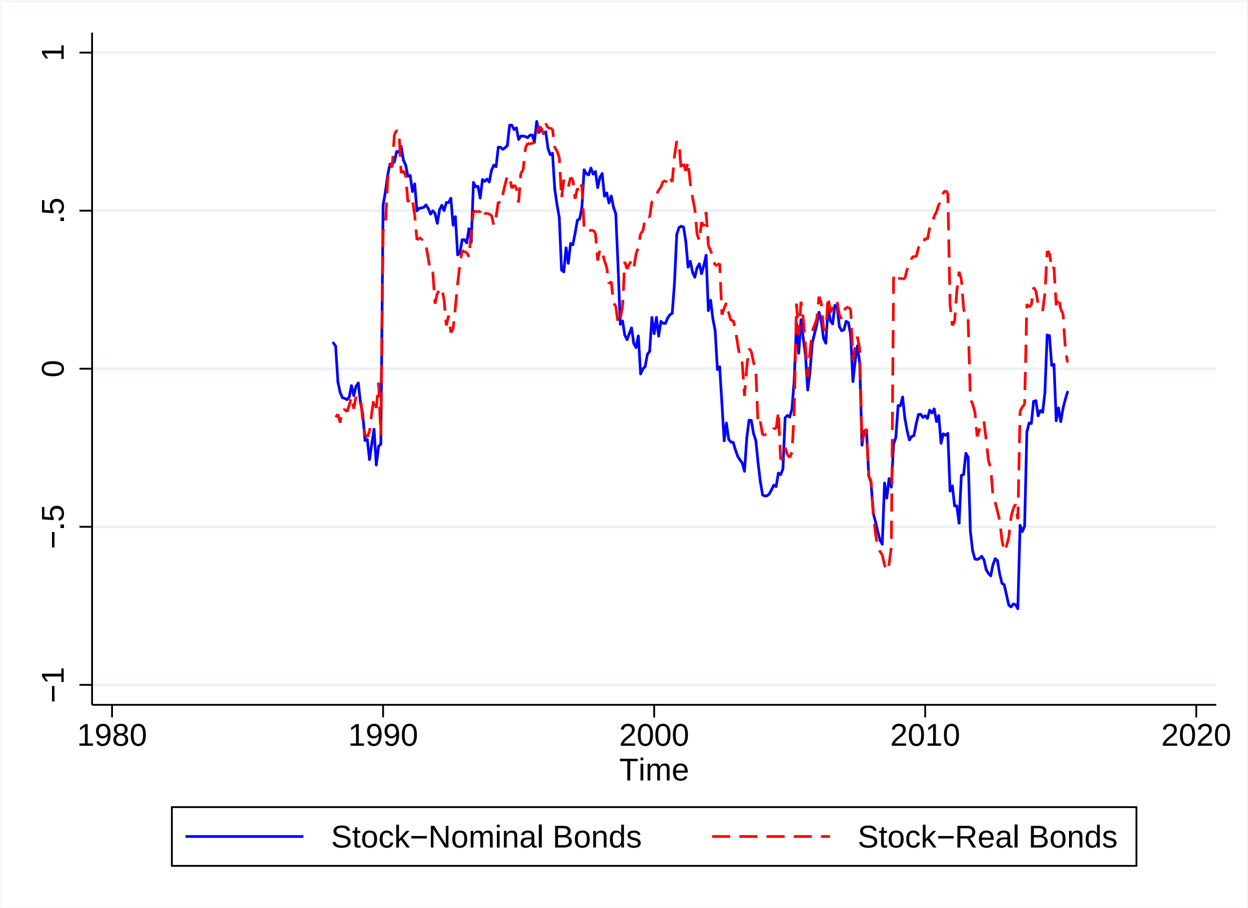
This figure displays correlations produced using monthly returns for nominal and real bonds in U.S. The estimate for month t is the sample correlation of the 25 returns for months $t-12$ through $t+12$.

Figure 3: Realized Stock-Bond Correlation for U.K.



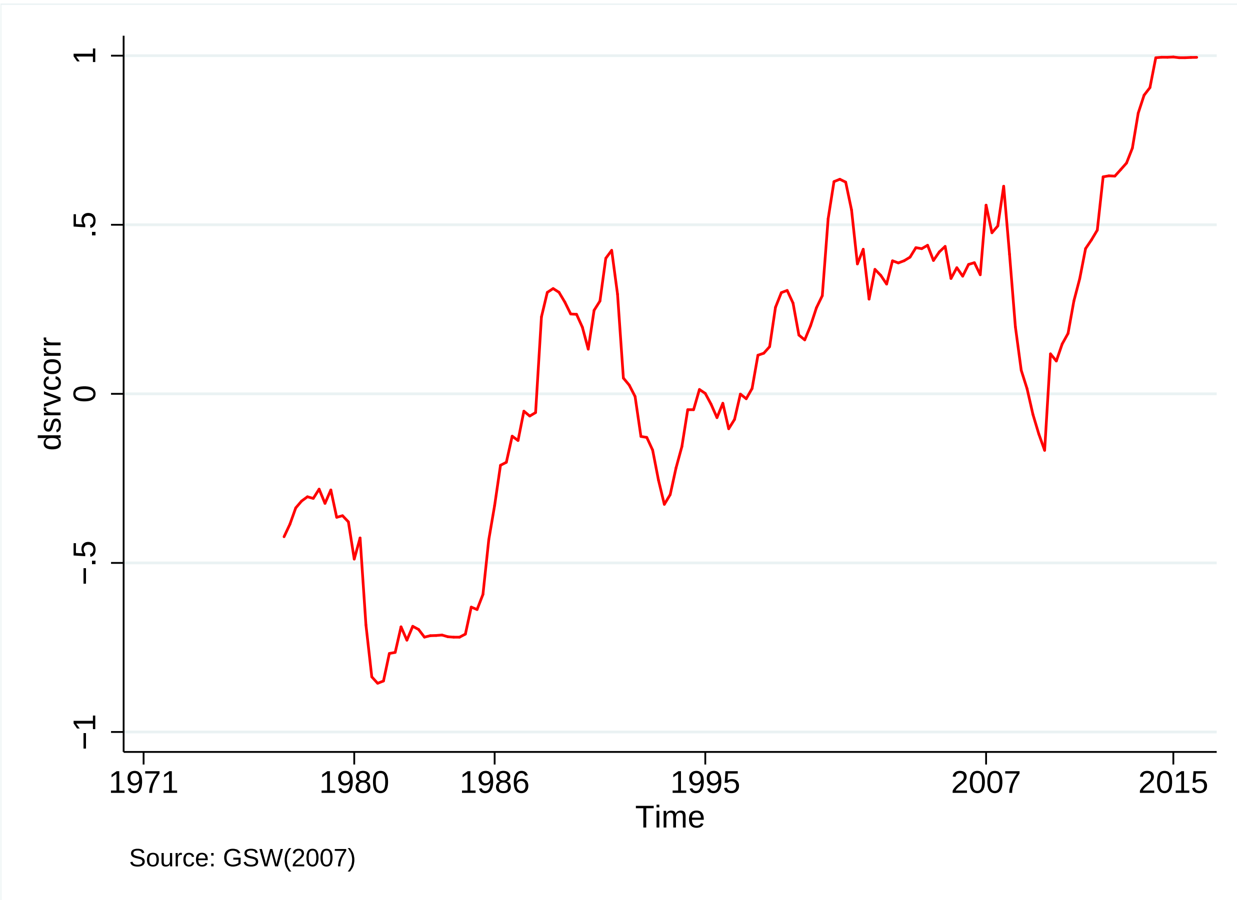
This figure graphs realized yearly correlations measured using daily returns for nominal and real bonds in UK. The data sample is from January 1986 to December 2015.

Figure 4: Moving Stock-Bond Correlation for U.K.



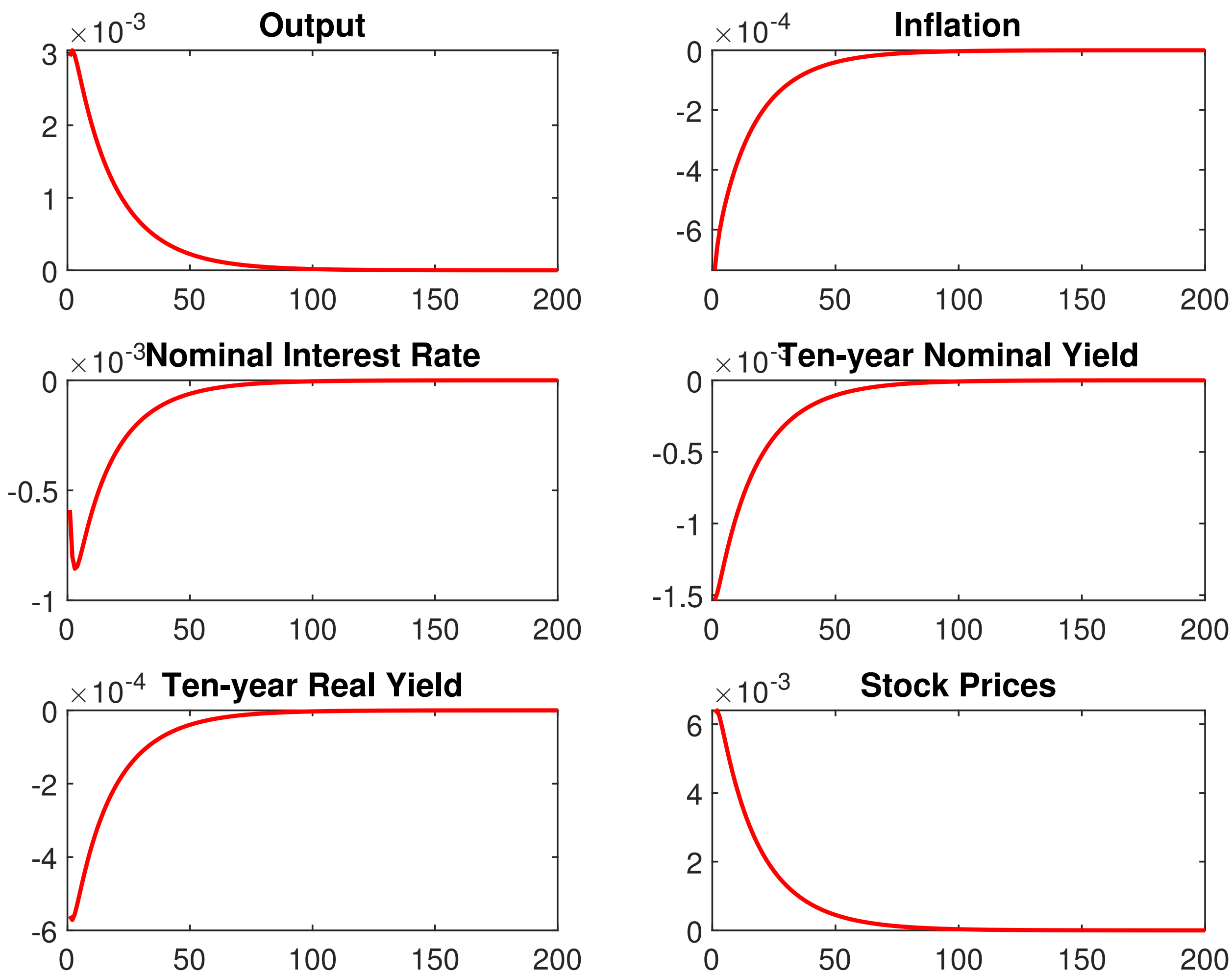
This figure displays correlations produced using monthly returns for nominal and real bonds in U.K. The estimate for month t is the sample correlation of the 25 returns for months $t-12$ through $t+12$.

Figure 5: Correlation between Slope Changes and Yields Changes



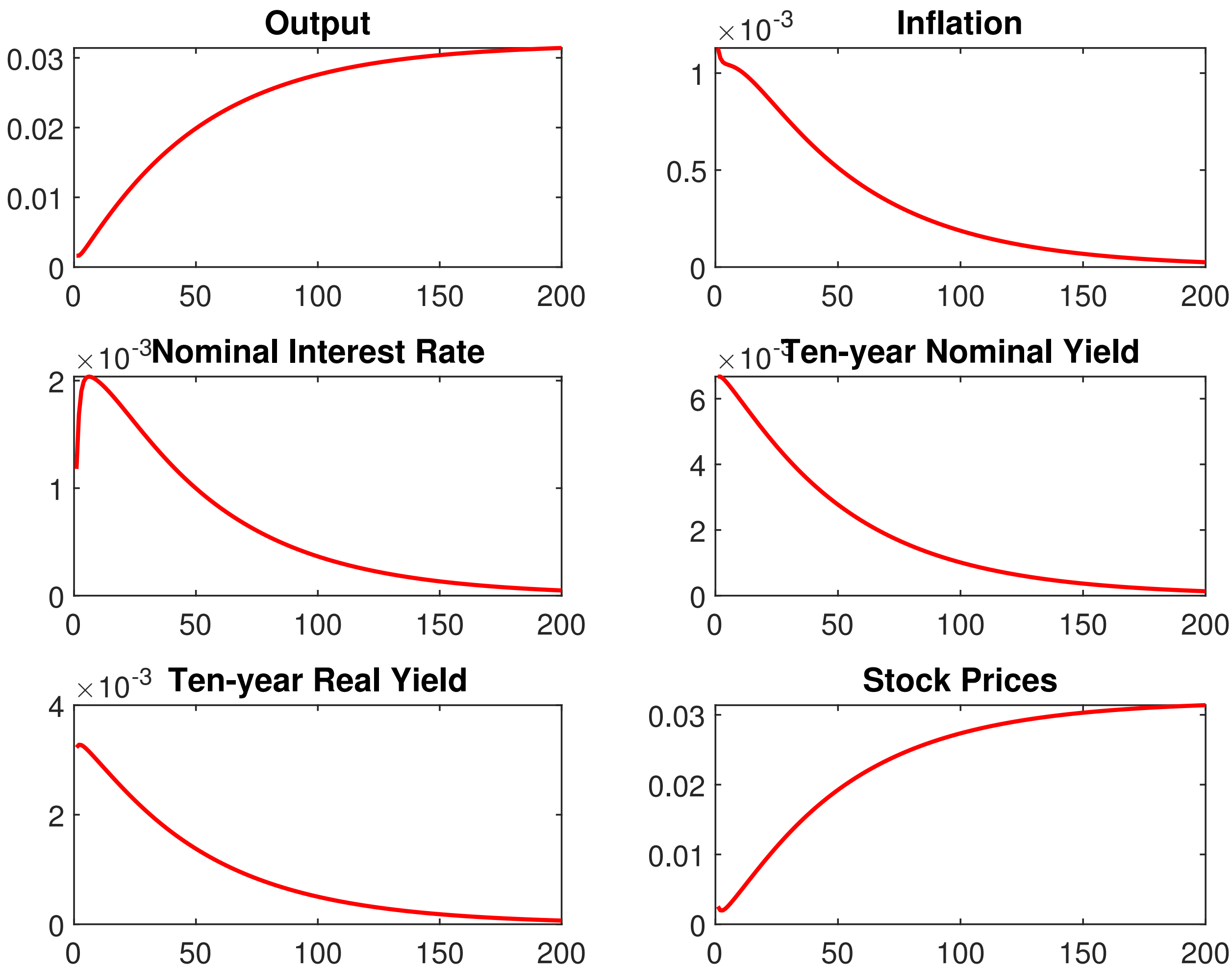
This figure graphs the 5-year moving quarterly correlations between changes in 5-year bond yield and changes in yield curve slope . The slope is measured by the 5-year yield less the three-month bill rate.

Figure 6: Impulse Response Functions for Level Shocks



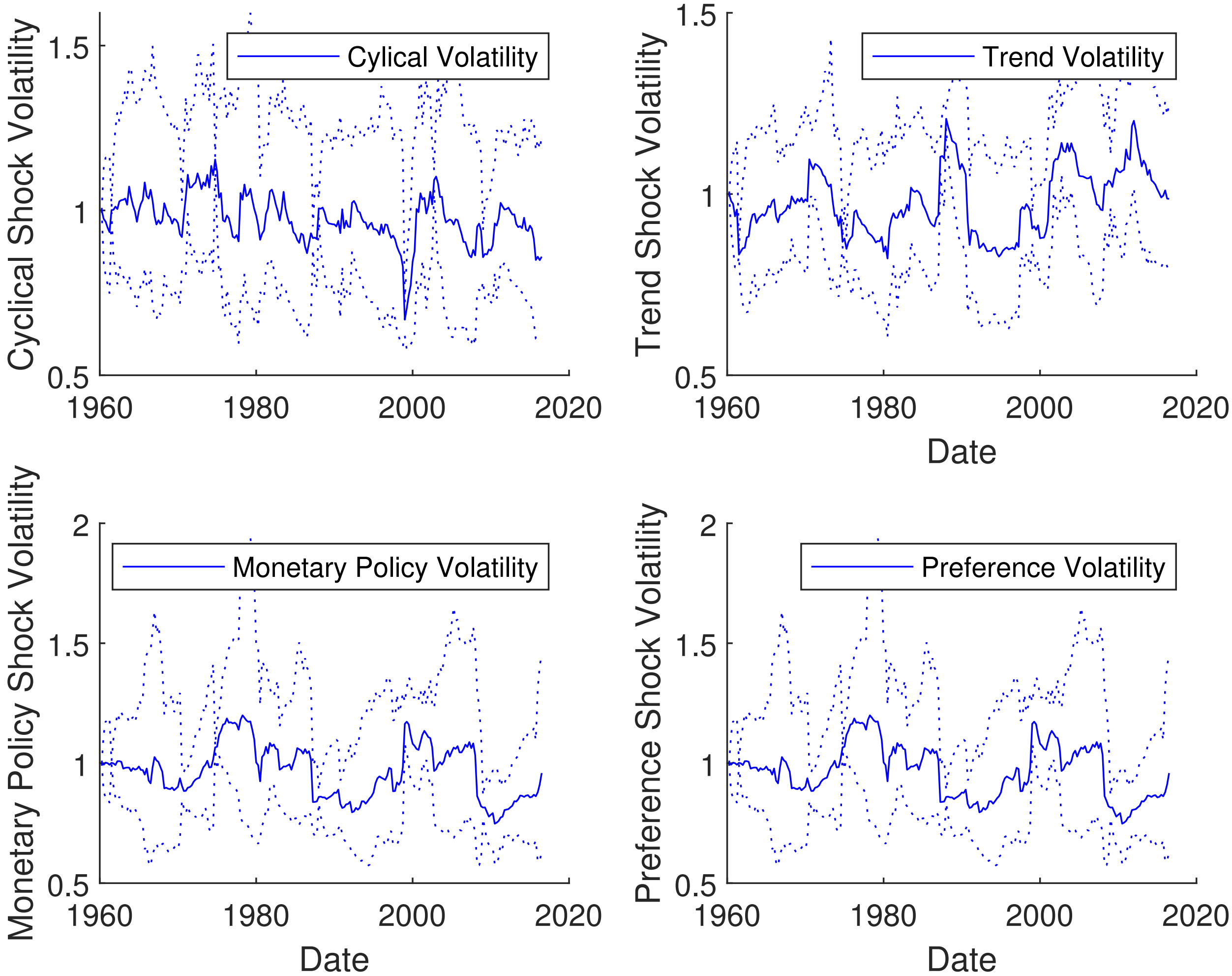
This figure shows average simulated impulse responses to a one standard deviation level technology shock for the output, inflation, the nominal interest rate, the nominal and real 10-year yields, and the stock prices.

Figure 7: Impulse Response Functions for Trend Shocks



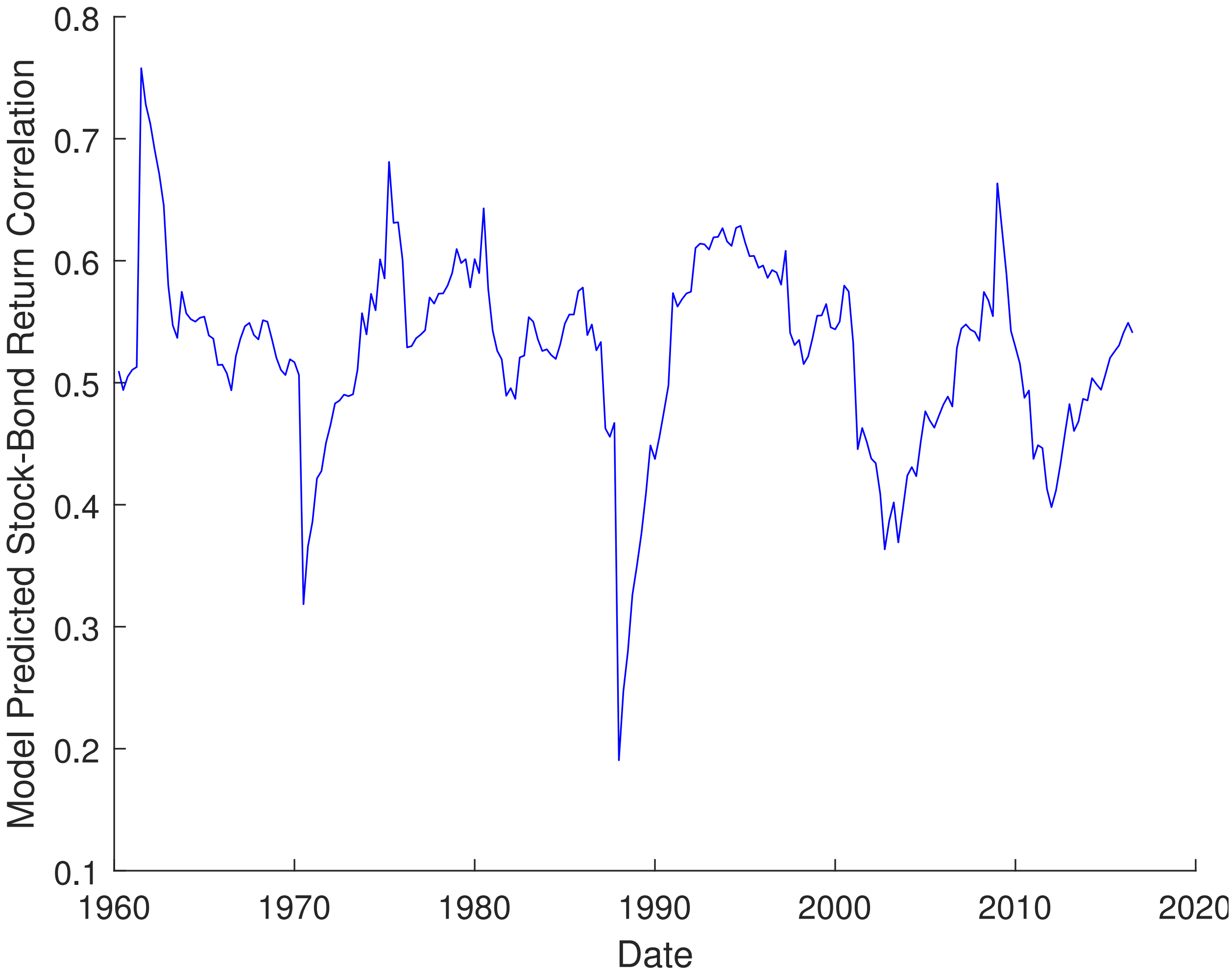
This figure shows average simulated impulse responses to a one standard deviation level technology shock for the output, inflation, the nominal interest rate, the nominal and real 10-year yields, and the stock prices.

Figure 8: Smoothed Volatility Dynamics



This figure plots the smoothed volatilities in percentage deviation from their means. The dashed lines represent 95 percent confidence intervals.

Figure 9: Model Predicted Stock-Bond Return Correlation



This figure plots the model predicted stock-bond return correlation.

Appendix A Data

A.1 U.S Stock-Bond Data

The stock data used for U.S is the return on S&P 500. The nominal bonds and real bonds data is from Gürkaynak et al. (2007) and Gürkaynak et al. (2010).

A.2 U.K Stock-Bond Data

The stock data used for U.K is the return on FTSE 100 index. FTSE index began on 3 January 1984 at the base level of 1000. It is a share index of the 100 companies listed on the London Stock Exchange with the highest market capitalization. The nominal and real bonds data is from the Bank of England at :
http://www.bankofengland.co.uk/statistics/Pages/yieldcurve/archive.aspx .

Appendix B Model Derivations

B.1 Price of a Utility Claim and the SDF Under Epstein-Zin Preferences

The stochastic discount factor (SDF) or the marginal rate of substitution of consumption between neighboring dates is

$$M_{t,t+1} \equiv \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} \tag{32}$$

By using the chain rule of derivatives, we have

$$\frac{\partial V_t}{\partial C_{t+1}} = \frac{\partial V_t}{\partial (\mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}} \frac{\partial (\mathbb{E}_t(V_{t+1}^{1-\gamma}))^{\frac{1}{1-\gamma}}}{\partial V_{t+1}} \frac{\partial V_{t+1}}{\partial C_{t+1}} \tag{33}$$

Combining it with

$$\frac{\partial V_t}{\partial \mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}} = \beta V_t^{\psi} \mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{-\psi}{1-\gamma}} \tag{34}$$

,

$$\frac{\partial \mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}}{\partial V_{t+1}} = V_{t+1}^{-\gamma} \mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{\gamma}{1-\gamma}} \tag{35}$$

$$\frac{\partial V_{t+1}}{\partial C_{t+1}} = (1-\beta)U_{C,t+1}\lambda_{t+1}V_{t+1}^{\psi}; \quad \frac{\partial V_t}{\partial C_t} = (1-\beta)U_{C,t}\lambda_t V_t^{\psi} \tag{36}$$

Then we have,

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}} \right]^{\psi-\gamma} \tag{37}$$

The total wealth is the discounted future value of aggregate consumption. The value of the total wealth $W_{U,t}$ is the lifetime-utility value, converted to real consumption units by dividing by the marginal lifetime-utility of a unit of consumption good. Formally,

$$W_{U,t} = \mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}} \left(\frac{\partial V_t}{\partial \mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}} \right) \left(\frac{\partial V_t}{\partial C_t} \right)^{-1} \tag{38}$$

$$= \mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}} \beta \mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{-\psi}{1-\gamma}} V_t^{\psi} (1-\beta)^{-1} V_t^{-\psi} C_t^{\psi} \lambda_t^{-1} \tag{39}$$

$$= \beta(1-\beta)^{-1} \mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{1-\psi}{1-\gamma}} C_t^{\psi} \lambda_t^{-1} \tag{40}$$

$$C_{t+1} + W_{U,t+1} = V_{t+1} \left(\frac{\partial V_{t+1}}{\partial C_{t+1}} \right)^{-1} = (1-\beta)^{-1} V_{t+1}^{1-\psi} C_{t+1}^{\psi} \lambda_{t+1}^{-1} \tag{41}$$

The gross return to the total is then

$$R_{W,t+1} = \frac{C_{t+1} + W_{U,t+1}}{W_{U,FFkt}} = \beta^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{\psi} \left(\frac{V_{t+1}}{\mathbb{E}_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}} \right)^{1-\psi} \left(\frac{\lambda_{t+1}}{\lambda_t} \right)^{-1} \tag{42}$$

Combining (37) and (42), we have

$$M_{t,t+1} = \left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\psi}\frac{\lambda_{t+1}}{\lambda_t}\right)^{\frac{1-\psi}{1-\gamma}}\left(R_{W,t+1}^{-1}\right)^{\frac{\gamma-\psi}{1-\psi}} \tag{43}$$

B.2 Labor Supply

It is also useful to note that household intratemporal optimality condition implies that

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{44}$$

Given the utility function specified in the paper, it translate into

$$w_t - p_t = \sigma c_t + \varphi n_t + (\chi + \Gamma_t)(1-\sigma) \tag{45}$$

Equation (44) can be interpreted as a competitive labor supply schedule, determining the quantity of labor supplied as a function of the real wage, given the marginal utility of consumption.

B.3 Optimal Price Setting and Inflation Dynamics

The optimality condition associated with the firm optimizing the price in period t is

$$\sum_{k=0}^{\infty}\theta^k\mathbb{E}_t\left\{M_{t,t+k}Y_{t+k|t}(P_t^* - \mathcal{M}\psi_{t+k|t})\right\} = 0 \tag{46}$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$ denotes the (nominal) marginal cost of firms resetting in period $t+k$ for a a firm which last reset its price in period t and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$. Letting $\Pi_{t,t+k} \equiv P_{t+k}/P_t$, it is useful to rewrite the equation above as

$$\sum_{k=0}^{\infty}\theta^k\mathbb{E}_t\left\{M_{t,t+k}Y_{t+k|t}\left(\frac{P_t^*}{P_{t-1}} - \mathcal{M}MC_{t+k|t}\Pi_{t-1,t+k}\right)\right\} = 0 \tag{47}$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ is the real marginal cost in period $t+k$ for a firm whose price was last set in period t.

A first-order Taylor approximation of the optimal price setting condition (47) around the zero inflation steady state yields, after some manipulation, (Detailed derivations can been found at the Chapter 3 of Galí (2009))

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t\{mc_{t+k|t} + p_{t+k}\}$$

where $mc_{t+k|t} \equiv \log MC_{t+k|t}$ is the (log) real marginal cost and $\mu = \log \mathcal{M}$ is the log of the desired markup. In the steady state, we could also have the approximate relationship between aggregate output, employment, and technology up to an order related to the dispersion of prices across firms (See Galí (2009)

$$n_t = \frac{1}{1-\alpha}[y_t - z_t - (1-\alpha)\Gamma_t - \alpha k_t] \tag{48}$$

Therefore, the (log) marginal cost per output for an individual firm that last resets its price in period t is given by

$$\begin{aligned} mc_{t+k|t} =& w_{t+k} - p_{t+k} - mpn_{t+k|t} \\ =& w_{t+k} - p_{t+k} - (z_{t+k} + \alpha k_{t+k} - \alpha n_{t+k|t} + (1-\alpha)\Gamma_{t+k} + \log(1-\alpha)) \end{aligned} \tag{49}$$

where mpn stands for marginal product per labor. The economy's average real marginal cost in period t is

$$\begin{aligned} mc_t =& w_t - p_t - mpn_t \\ =& w_t - p_t - (z_t + \alpha k_t - \alpha n_t + (1-\alpha)\Gamma_t + \log(1-\alpha)) \end{aligned} \tag{50}$$

Thus the following relation holds between firm-specific and economy-wide marginal costs:

$$\begin{aligned} mc_{t+k|t} =& mc_{t+k} + \alpha(n_{t+k|t} - n_{t+k}) \\ =& mc_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ =& mc_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k}) \end{aligned} \tag{51}$$

where the second equality follows from (48) and the third equality results from combining demand schedule and the good market clearing condition.

Substituting (51) into (47) and rearranging terms yields

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t\{p_{t+k} - \Theta\hat{\mu}_{t+k}\} \tag{52}$$

where $\hat{\mu}_t \equiv \mu_t - \mu$ is the deviation between the average and desired markups, with $\mu_t \equiv p_t - \log\psi_t = -mc_t$ and $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$. The above expression for p_t^* can be rewritten as a recursive equation:

$$p_t^* = \beta\theta\mathbb{E}_t\{p_{t+1}^*\} + (1-\beta\theta)(p_t - \Theta\hat{\mu}_t) \tag{53}$$

As shown by Galí (2009), the above environment implies that the aggregate price dynamics are described by the equation

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{54}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross rate of inflation between $t-1$ and t and P_t^* is the price set in period t by firms reoptimizing their price in that period. A log-linear approximation to the aggregate price index around the steady state yields

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{55}$$

or, equivalently, after rearranging terms:

$$p_t = \theta p_{t-1} + (1-\theta)p_t^* \tag{56}$$

Finally, combining (55) and (53) yields the inflation equation

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} - \lambda \hat{\mu}_t \tag{57}$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta \tag{58}$$

Appendix C Estimation

The section describes the details of estimating the model with stochastic volatility. The state-space representation of the model is summarized in Equation (6), (30) and (31) as

$$z_t = \Phi(\kappa)z_{t-1} + v(\sigma_t)\epsilon_t \tag{59}$$

$$x_t = \mu_x(\kappa) + \mathrm{T}(\kappa)s_{t-1} + \omega_t \tag{60}$$

where Σ_ω is measurement error, σ_t follows autoregressive processes in Assumption 2.

C.1 Particle Filter

Let $x_{1:T}$ denotes the observables from period 1 till T. Because the likelihood function of the model $p(x_{1:T})$ is not known in a closed form, The estimation method uses a particle filter to approximate the likelihood. The implementation of the particle filter is based on the Algorithm 13 in Herbst and Schorfheide (2015). The particle filter uses a swarm of particles $\{s_j, W_j\}_{j=1}^{T}$to approximate the likelihood, where s_t^j are particle values and the W_t^j are the particle weights. The conditional expectation of $h(s_t)$ is approximated by a weighted average of the (transformed) particles $h(s_t^j)$.

Algorithms 1:

1. Initialization. Draw the initial particles from the distribution $s_0^j \sim p(s_0)$, $j = 1, ..., M$.

2. Recursion. For $t = 1, ..., T$:

   (a) **Forecasting** s_t: Draw $\hat{s}_t^j$ from density $p(\hat{s}_t^j|s_{t-1}^j)$. An approximation of $\mathbb{E}[h(s_t)|x_{1:t-1}]$ is given by

$$\mathbb{E}[h(s_t)|x_{1:t-1}] = \frac{1}{M}\sum_{j=1}^{M} h(\hat{s}_t^j) \tag{61}$$

   (b) **Forecasting** x_t: The predictive density $p(x_t|x_{1:t-1})$ can be approximated by the average of incremental weight $p(x_t|\hat{s}_t^j)$

$$p(x_t|x_{1:t-1}) = \frac{1}{M}\sum_{j=1}^{M} p(x_t|\hat{s}_t^j) \tag{62}$$

   (c) **Updating.** Define the normalized weights

$$\hat{w}_t^j = \frac{p(x_t|\hat{s}_t^j)}{\frac{1}{M}\sum_{j=1}^{M} p(x_t|\hat{s}_t^j)} \tag{63}$$

   (d) **Selection.** Resample the particles via multinomial resampling. Let $\{s_t^j\}_{j=1}^M$ denote M *iid* draws from a multinominal distribution characterized by support points and weights $\{\hat{s}_t^j, \hat{w}_t^j\}$. An approximation of $\mathbb{E}[h(s_t)|x_{1:t}, \kappa]$ is given by

$$\mathbb{E}[h(s_t)|x_{1:t}] = \frac{1}{M}\sum_{j=1}^{M} h(s_t^j)w_t^j \tag{64}$$

3. Likelihood Approximation. The approximation of the log likelihood function is given by

$$\log p(x_{1:T}|\kappa) = \sum_{t=1}^{T} \log\left(\frac{1}{M}\sum_{j=1}^{M} p(x_t|s_t^j)\right) \tag{65}$$

In this version of the particle filter, the time t particles are generated based on the time $t-1$ particles by simulating the state-transition equation forward. The particle filter weights are then updated based on the likelihood of the observation x_t under the s_t^j particle, $p(x_t|s_t^j)$. The more accurate the prediction of x_t based on s_t^j, the larger the density $p(x_t|s_t^j)$, the larger the density $p(x_t|s_t^j)$, and the larger the relative weight that will be placed on particle j.

The selection step is included in the filter to avoid a degeneracy of particle weights. While it adds additional noise to the Monte Carlo approximation, it simultaneously equalizes the particle weights, which increases the accuracy of subsequent approximations. In the absence of the selection step, the distribution of particle weights would become more uneven from iteration to iteration. The selection step does not have to be executed in every iteration. For instance, in practice, users often apply a threshold rule according to which the selection step is executed whenever the following measure falls below a threshold, e.g., 25% or 50% of the nominal number of particles:

$$E\hat{S}S_t = M/\left(\frac{1}{M}\sum_{j=1}^{M}(\hat{w}_t^j)^2\right)$$

The effective sample size $E\hat{S}S_t$ (in terms of number of particles) captures the variance of the particle weights. It is equal to M if $\tilde{W}_t^j = 1$ for all $j = 1$ and equal to 1 if one of the particles has weight M and all others have weight 0.

C.2 Smoother

After the filter is performed on the entire data set, I have an approximate representation of $p(s_t|x_{1:t})$ for each time step $t = 1, ...T$, consisting of weighted particles $\left\{s_t^j, w_t^j\right\}$, $j = 1, ...M$, where M is the number of particles used for approximation. I employ the backward-smoothing routine suggested by Godsill et al. (2004) to draw from the smoothing density $p(s_{1:T}|x_{1:T};\kappa)$ to get a historical distribution of the latent/hidden states. I build on the factorization

$$p(s_{1:T}|x_{1:T}) = p(s_T|x_{1:T})\prod_{t=1}^{T-1} p(s_t|s_{t+1:T}, x_{1:T}) \qquad (66)$$

where, using the Markovian assumptions of the model

$$p(s_t|s_{t+1:T}, x_{1:T}) = p(s_t|s_{t+1}, x_{1:t}) \tag{67}$$

$$= \frac{p(s_t|x_{1:t})p(s_{t+1}|s_t)}{p(x_{t+1}|x_{1:t})} \tag{68}$$

$$\propto p(s_t|x_{1:t})p(s_{t+1}|s_t) \tag{69}$$

Since the forward filtering generates an approximation to $p(s_t|x_{1:t})$, we immediately obtain the modified particle approximation

$$p(s_t|s_{t+1}, x_{1:T}) \approx \sum_{i=1}^{M} w^j_{t|t+1}\delta_{s^j_t}(s_j) \tag{70}$$

with modified weights

$$w^j_{t|t+1} = \frac{w^j_t p(s_{t+1}|s^j_t)}{\sum_{j=1}^{M} w^j_t p(s_{t+1}|s^j_t)} \tag{71}$$

where δ is the Dirac delta function and w^j_t is a weight attached to particle s^j_t. The revised particle filter can now be used to generate states successively in the reverse-time direction, conditioning on future states. Specifically, given a random sample $\{s^j_{t+1}\}$, $j = 1, ..., M$ drawn approximately from $p(s_{t+1:T}|y_{1:T})$, take one step back in time and sample s^i_t from $p(s_t|s_{t+1}, x_{1:T})$. The pair $\{(s^j_{t+1}, s^i_t)\}$ is then approximately a random realization of $p(s_{t:T}|x_{1:T})$. Repeating this process sequentially over time produces the following general "smoother" algorithm:

Algorithms 2:

1. Initialization: Draw M particles $\left\{s^j_T\right\}$ from $p(s_T|x_{1:T}; \kappa)$

2. For $t = T - 1$ to 1:

   * Calculate $w^j_{t|t+1} \propto w^j_t \, p(x^j_{t+1}|x^i_t)$ for each $i = 1, ...M$.

   * Resample: choose $\{s_t\}$, $i = 1, ...M$ with probability $w^i_{t|t+1}$.

3. $s_{1:T} = \{(s_1, s_2, ..., s_T)\}$ is an approximate realization from $p(s_{1:T}|x_{1:T})$

As the number of particles goes to infinity, the simulated conditional distribution of states converges to the unknown true conditional density.

C.3 Random Walk Metropolis Hasting Algorithm

To gain posterior sampler of the parameters of the model, the particle filter is embedded into a standard random-walk Metropolis-Hasting algorithm described by Herbst and Schorfheide (2015) (Chapter 9). The algorithm proceeds as

Algorithms 3:

1. For $i = 1 : N$. Draw the parameter vector ν from the density $q(\nu|\kappa_{i-1})$
2. Set $\kappa_i = \nu$ with probability $\alpha(\nu|\kappa_{i-1}) = \frac{p(x|\nu)p(\nu)}{p(x|\kappa_{i-1})p(\kappa_{i-1})}$ and $\kappa_i = \kappa_{i-1}$ otherwise. The likelihood function $p(x|\kappa)$ is approximated by the particle filter.

Iterating over steps 1 to 2, we can - after a suitable burn-in-period - obtain samples form the desired posterior distribution, which is the invariant distribution of the resulting Markov Chain. In our case, a burn-in of 2500 proved sufficient.

C.4 Empirical Estimation Using Only Productivity Series

Historically, U.S labor productivity growth (defined as output per hour worked) in the business sector has varied greatly. Strong growth rate of 3.3% in the period of 1947-1973 was followed by a sharp slowdown to 1.6% in the two decades that followed. The information and communication technology (ICT) boom in period $1996 - 2003$ led to the “productivity miracle”, when labor productivity growth doubled. As the gains from the ICT boom had largely been reaped, productivity growth slowed down to 1.9% in the pre-crisis years (2004-2007). Labor productivity growth has been moderate since the crisis.

In this section, I estimate the dynamics of productivity allowing for stochastic volatility as specified in the model section of Assumption 2 using only the productivity growth series. The average of the growth rate μ_g is set to be 0.8% such that the model's long-term average of productivity growth $(1 - \alpha) * \mu_g$ matches the historical average of productivity growth. There are in total 8 parameters to be estimated for the cyclical and trend component. Prior distributions for these parameters are the same as in Section 5.1. The prior distributions are summarized in the following table.

After the estimation, I find that the the median persistent parameters ρ_z and ρ_g for the level of the cyclical and trend component is 0.64 and 0.77. These persistence parameters at the quarterly frequency suggest that the cyclical and trend components are moderately persistent. However, we should treat these point estimates with caution because corresponding confidence intervals are not very small. The 5% and 95% percentiles are 0.52 and 0.88 for the cyclical component, and 0.65 and 0.91 for the trend component. Given the length for the quarterly productivity series that are available from 1961, pinning down these parameters with small confidence intervals is an economically challenging task. Therefore, it is necessary to use financial data to increase the precision of these estimates. These numbers echo the estimates of the persistence parameter for the long-run component of quarterly consumption growth by Schorfheide et al. (2018). They find that the posterior median estimates of persistence parameters are about 0.65 estimated using. quarterly consumption growth.

As for the persistence parameters for the volatility, the posterior median of $\rho_{\sigma,z}$ and $\rho_{\sigma,g}$ are 0.91 and 0.75. The 5% and 95% percentiles is 0.83 and 0.96 for $\rho_{\sigma,z}$, while the percentiles are 0.58 and 0.97 for the $\rho_{\sigma,g}$. Therefore, volatility of the trend component is harder to pinning down relative to the one in the cyclical component.

The posterior median for the log of the cyclical and trend volatility σ_z, σ_g is -5.0 and -5.9 respectively, suggesting the trend shock volatility is smaller than the cyclical shock volatility.

Table 4: Prior and posterior distributions of volatility process parameters estimated using only productivity

Parameter	Distribution	Prior			Posterior		
		5%	50%	95%	5%.	50%	95%
Cyclical Shock							
ρ_z	U	-0.90	0	0.90	0.52	0.64	0.88
$\rho_{\sigma z}$	U	-0.90	0	0.90	0.83	0.91	0.96
σ_z	N	-8	-4	0	-5.24	-5.02	-4.76
$\eta_{\sigma z}$	IG	0.06	0.15	0.49	0.10	0.16	0.19
Trend Shock							
ρ_g	U	-0.90	0	0.90	0.65	0.77	0.91
$\rho_{\sigma g}$	U	-0.90	0	0.90	0.58	0.75	0.91
σ_g	N	-9	-5	-1	-6.51	-5.91	-5.57
$\eta_{\sigma g}$	IG	0.06	0.15	0.49	0.06	0.12	0.20

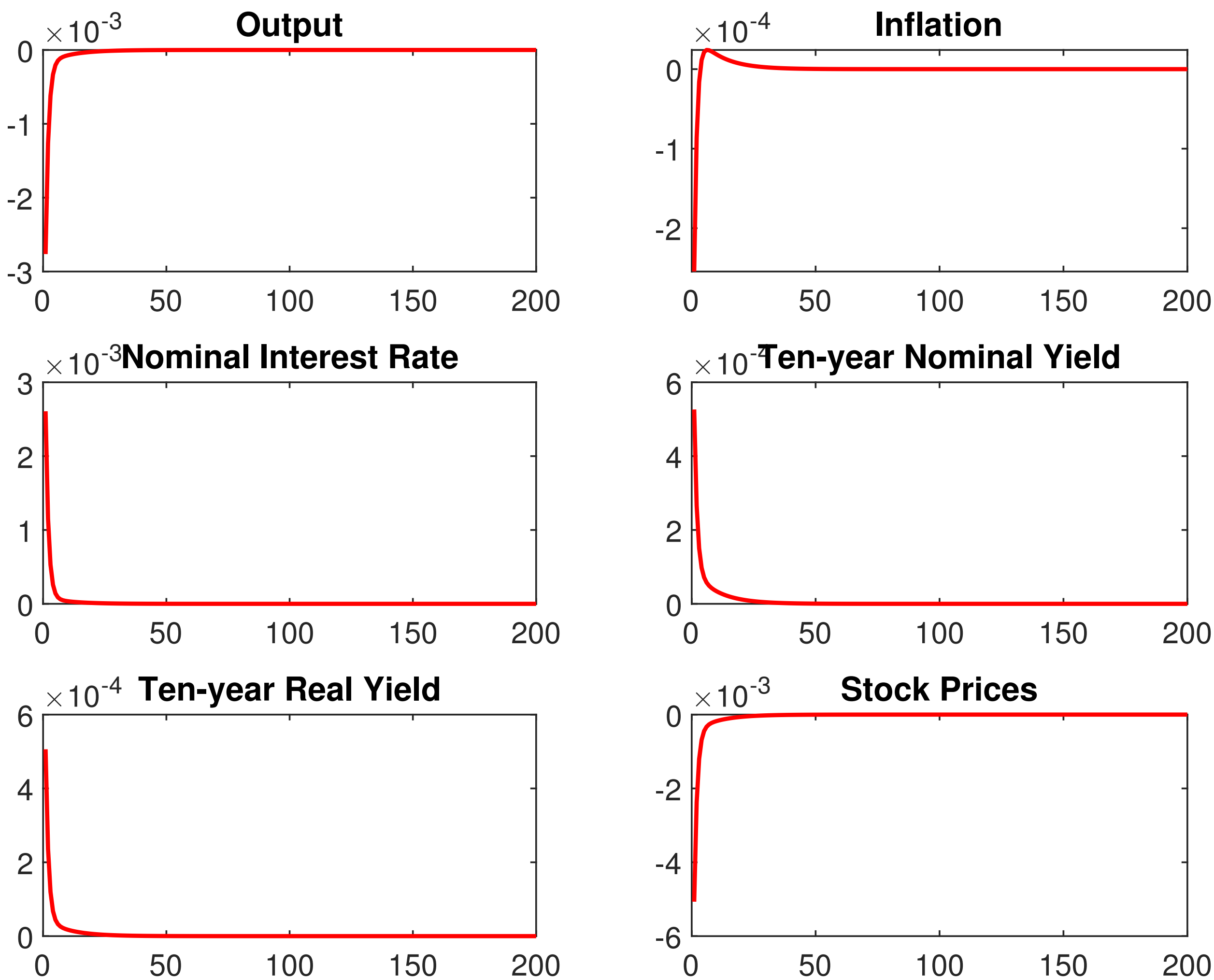
This table reports the prior and posterior distribution of parameters from the estimation of the model. The table summarizes distributions of volatility parameters of cyclical component and distributions of the trend component. There are eight parameters estimated. ρ_z and ρ_g denote the persistence of the cyclical and trend component of productivity. $\rho_{\sigma z}$ and $\rho_{\sigma g}$ denote the persistence of the volatility process. σ_z and σ_g denote the steady state log standard deviation of the cyclical shock and trend shock. $\eta_{\sigma z}$ and $\eta_{\sigma g}$ denote the standard deviation of shocks to the volatility process. U, N, and IG denote normal, uniform and inverse gamma distribution respectively.

C.5 Impulse Responses of Monetary Policy and Preference Shocks

This section presents the impulses of variables to monetary policy and preference shocks. Figure 10 shows responses of output, inflation, nominal interest rate, the yield for 10-year nominal and real bonds and stock prices to a monetary shock. A positive monetary policy shock is contractionary and acts as a negative impulse to output and inflation. Long-term real interest rates increase as people expect the economy to go back to long-run level. So monetary policy shocks lead to positive stock-bond correlations.

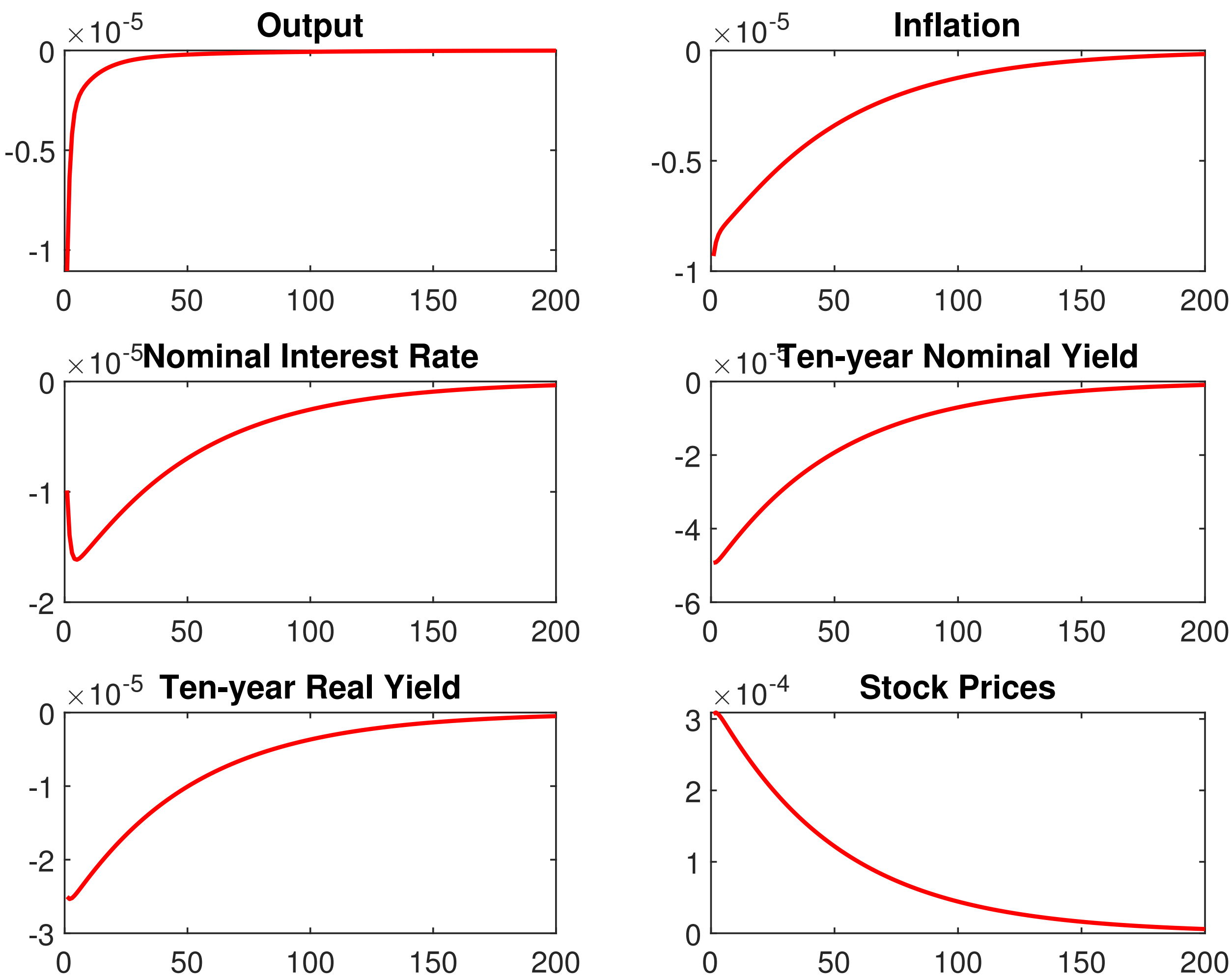
Figure 11 shows responses of output, inflation, nominal interest rate, the yield for 10-year nominal and real bonds and stock prices to a preference shock. A positive preference shock makes agents more patient and acts a negative impulse to output and inflation and interest rates falls. Stock and bond prices increase after a positive preference shock.

Figure 10: Impulse Response Functions for A Monetary Policy Shock



This figure shows average simulated impulse responses to a one standard deviation monetary policy shock for the output, inflation, the nominal interest rate, the nominal and real 10-year yields, and the stock prices.

Figure 11: Impulse Response Functions for A Preference Shock



This figure shows average simulated impulse responses to a one standard deviation preference shock for the output, inflation, the nominal interest rate, the nominal and real 10-year yields, and the stock prices.