The Short-Run and Long-Run Components of Idiosyncratic Volatility and Stock Returns

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Abstract

To capture the dynamics of idiosyncratic volatility of stock returns over different horizons and investigate the relationship between idiosyncratic volatility and expected stock returns, this paper develops and estimates a parsimonious model of idiosyncratic volatility consisting of a short-run and a long-run component. The conditional short-run and long-run components are found to be positively and negatively related to expected stock returns, respectively. The positive relation between the short-run component and stock returns may be caused by investors requiring compensation for bearing idiosyncratic volatility risk when facing trading frictions and hold under-diversified portfolios. The negative relationship between the long-run component and stock returns may reflect the fact that stocks with high long-run idiosyncratic volatility are less exposed to systematic risk factors, and hence earn lower returns. Moreover, the low risk exposure of stocks characterized by high idiosyncratic volatility lends support to real-option-based mechanisms to explain this negative relation. In particular, the systematic risk of a firm with abundant growth options crucially dependent upon the risk exposure of these options. The value of growth options could rise significantly due to convexity when the increase in idiosyncratic volatility occurs over long horizons. And growth options' systematic risk could fall because the relative magnitude of their value in relation to systematic risk factors decreases.

Keywords: idiosyncratic volatility, short-run (long-run), cross-sectional stock returns, risk factors, real options

JEL Code: G10, G12, G13, G17, G31

1 Introduction

The question of whether a stock's expected return depends on idiosyncratic volatility has been a central theme of the asset pricing literature. In an influential paper, Ang et al. (2006) present evidence that stocks with high realized idiosyncratic volatility have anomalously low returns in the subsequent month. This phenomenon is challenging to interpret because traditional asset pricing theories predict no relation between idiosyncratic volatility and expected returns when investors are well-diversified and markets are complete and frictionless, or a positive relation when investors don't hold well diversified portfolios and face trading frictions (for example, Merton 1987).

Ang et al. (2006) define idiosyncratic volatility as the standard deviation of the residuals from the Fama and French (1993) (hereafter FF-3) model estimated using daily returns from the previous month. Because idiosyncratic volatility is time-varying, lagged idiosyncratic volatility may not be a good proxy for the conditional idiosyncratic volatility. However, the dynamics of conditional idiosyncratic volatility and its relationship with cross-sectional stock returns remain unclear. Fu (2009) constructs measures of conditional idiosyncratic volatility using exponential generalized autoregressive processes (EGARCH) and instead finds a strong positive relationship between conditional idiosyncratic volatility and average returns. Nonetheless, Guo et al. (2014) caution that the EGARCH approach used by Fu (2009) may be subject to substantial look-ahead bias. Once the look-ahead bias is addressed, Fink et al. (2012) also do not find a positive relationship between idiosyncratic volatility and expected returns using the EGARCH model. Finally, Ang et al. (2009) demonstrate that lagged realized idiosyncratic volatility possesses strong explanatory power for 1-month-ahead realized idiosyncratic volatility, indicating that lagged realized volatility may serve as a useful proxy for the conditional idiosyncratic volatility. However, they do not provide a structural model for the dynamics of idiosyncratic volatility.

This paper mainly makes three contributions to the literature. First, I document empirical evidence that idiosyncratic volatility decays quickly between one and three months, while persisting over longer horizons. To capture the dynamics of idiosyncratic volatility over short and long horizons, I develop a parsimonious model of idiosyncratic volatility featuring two components differing in persistence. This modeling approach is in line with the work of Adrian and Rosenberg (2008), Corsi (2009), and Christoffersen et al. (2008). The more persistent component is termed the long-run component and could be modeled as containing a unit root. The other component is referred to as the short-run component and is less persistent. I use both portfolio analysis and Fama and MacBeth (1973) regressions to investigate the cross-sectional relationship between these two components and stock returns. Results from both types of methods indicates a significant negative relation between the conditional long-run idiosyncratic volatility and expected returns, and a significant positive relation between the conditional short-run idiosyncratic volatility and expected returns. Therefore, accounting for the dynamics of idiosyncratic volatility over short and long horizons is crucial to the measurement of conditional idiosyncratic volatility and understanding of the relationship between idiosyncratic volatility and expected stock returns.

Moreover, the return spread between the lowest and highest quintile portfolio sorted by the conditional long-run idiosyncratic volatility is correlated with the return spread sorted by the realized idiosyncratic volatility, with a coefficient of 0.95. And the averages of these return spreads are also quantitatively close, with -0.79% per month for the realized volatility and -0.73% for the conditional long-run idiosyncratic volatility. This finding suggests that the negative relationship between realized idiosyncratic volatility and stock returns is limited to the long-run idiosyncratic volatility and provides a new dimension for investigating potential mechanisms behind this negative relationship.

Second, this paper provides empirical evidence suggesting that the cross-sectional relationship between conditional long-run idiosyncratic volatility and stock returns may be risk-driven, while the relationship between conditional short-run idiosyncratic volatility and stock returns is not. I include three different predictive horizons (1, 12, and 24 months) in the portfolio analysis, and find that the predictive relationship between conditional long-run idiosyncratic volatility and expected returns holds for the 1-, 12- and 24-month horizons. In contrast, the predictive relationship of conditional short-run idiosyncratic volatility only holds for the 1-month horizon. This finding highlights that there are persistent variations in expected returns that are negatively related to conditional long-run idiosyncratic volatility. As Cochrane (1999) explains, if predictability reflects risk, it is likely to persist. Therefore, a risk-based explanation may be an effective means of explaining the persistent negative relationship between the conditional long-run volatility and expected returns, whereas the positive relationship between the conditional short-run idiosyncratic volatility and expected stock returns may not be driven by exposure to systematic risk factors.

Furthermore, I investigate whether the difference in portfolio returns sorted by the short-run and long-run components of idiosyncratic volatility might be explained by exposure to systematic risk factors. The return difference in portfolios sorted by the conditional short-run idiosyncratic volatility is not found to be correlated with common systematic risk factors. This lack of correlations with systematic risk factors is direct evidence against risk-based explanations for the predictability of short-run idiosyncratic volatility. The positive relationship between conditional short-run idiosyncratic volatility and stock returns may arise because investors require compensation for bearing idiosyncratic risk when facing trading frictions in short horizons, and hence hold under-diversified portfolios (Merton (1987)). In contrast, the difference in portfolio returns sorted by the conditional long-run idiosyncratic volatility comoves with systematic risk factors. In particular, I find that portfolios with high idiosyncratic volatility are less exposed to the profitability factor in the five-factor model of Fama and French (2015).

Third, the finding that the negative relation between realized idiosyncratic volatility and stock returns is limited to the long-run idiosyncratic volatility lends support to real-option-based mechanisms as means of explaining the low risk exposure to systematic risk factors of stocks with high long-run idiosyncratic volatility. Real-option-based theories, following Berk et al. (1999) and Carlson et al. (2004), model the value of firms deriving from assets in place and growth options. Firms could exploit valuable investment opportunities by making irreversible investments. Bhamra and Shim (2017) introduce stochastic idiosyncratic cash flow risk into a real-option model with growth options to explain the negative relationship between idiosyncratic volatility and stock returns. For a firm with abundant growth options, its systematic risk crucially depends upon the risk exposure of such options. When idiosyncratic volatility increases, the value of growth options could rise significantly because of convexity. But growth options' exposure to systematic risk factors could fall, due to the decrease in the relative magnitude of the value of options related to systematic risk.

I also outline a model similar to that of Bhamra and Shim (2017) in the Appendix to illustrate this mechanism.

In addition, this real-option-based mechanism highlights the importance of long-run idiosyncratic volatility in explaining the negative relationship between idiosyncratic volatility and stock returns. The rise in growth option values could be pronounced when the increase in idiosyncratic volatility is over long horizons and there is a possibility of waiting to invest. The impact of short-run variations of volatility on option values could be limited. Therefore, only the persistent part of idiosyncratic volatility, i.e., long-run idiosyncratic volatility, is negatively related to cross-sectional stock returns.

The remainder of this paper is organized as follows. Section 2 describes how to measure the idiosyncratic volatilities of stocks and decompose them into short-run and long-run components. Section 3 explores the cross-sectional relationship between conditional short-run and long-run idiosyncratic volatility and stock returns using portfolio analysis. Section 4 examines such relationships via cross-sectional regressions. Section 5 investigates risk exposures of stocks with different levels of idiosyncratic volatility and discusses underlying mechanisms behind the cross-sectional relationship between idiosyncratic volatility and stock returns. In particular, the discussion sheds light on the implications of long-run idiosyncratic volatility for mechanisms behind the negative relationship between idiosyncratic volatility and stock returns. Section 6 concludes.

2 Estimating Idiosyncratic Volatilities

In this section, I describe the data and methods used to estimate idiosyncratic volatilities.

2.1 Data

My dataset includes monthly and daily return data on stocks traded in the NYSE, AMEX, and NASDAQ return files from the Center for Research in Security Prices (CRSP). The accounting variables are from COMPUSTAT's annual industrial files of income-statement and balance-sheet data.

The CRSP returns cover NYSE and AMEX stocks until 1973 when NASDAQ returns also come on line. The COMPUSTAT data covers the period from 1963 to 2017. The 1963 start date reflects the fact that the book value of common equity (COMPUSTAT item 60) is not generally available prior to 1962. More importantly, COMPUSTAT data from earlier years have a serious selection bias: the pre-1962 data are tilted toward big, historically successful firms.

The procedures below are standard in the literature following Fama and French (1992). To ensure that the accounting variables are known before the returns they are used to explain, I match the accounting data for all fiscal year ends in calendar year $t-1$ with the returns for July of year t to June of year $t+1$. The 6-month (minimum) gap between fiscal year end and the return tests is conservative. I use a firm's market equity at the end of December of year $t-1$ to compute its book-to-market ratio for year $t-1$.

2.2 Idiosyncratic Volatility Definition

Following Ang et al. (2006) and Bali and Cakici (2008), I concentrate on idiosyncratic volatility defined and measured relative to the Fama and French (1993) three-factor (FF-3) model.[1] Specifically, I consider the following specification for each firm at each month:

$$r^i_{t,d} = \alpha^i_t + \beta^i_{MKT} MKT_{t,d} + \beta^i_{SMB} SMB_{t,d} + \beta^i_{HML} HML_{t,d} + \sigma^i_t \epsilon^i_{t,d} \tag{1}$$

where for day d in month t, $r^i_{t,d}$ is stock i's excess return, $MKT_{t,d}$ is the market excess returns, $SMB_{t,d}$ and $HML_{t,d}$ capture size and book-to-market effects, respectively. The residuals $\eta^i_{t,d} \equiv \sigma^i_{t,d}\epsilon^i_{t,d}$ are the idiosyncratic risk for month t. I define the idiosyncratic volatility of stock returns for firm i in month t as

$$v^i_t = \sigma^i_t \sqrt{N_m} \tag{2}$$

[1] I also consider defining the idiosyncratic volatility relative to the Carhart (1997) four-factor model and Fama and French (2015) five-factor model. The negative relationship between lagged realized volatility and average stock returns also holds. These results are available upon request.

where N_m is the number of trading days in month t for firm i. It is useful to note that the idiosyncratic volatility v_t^i is the daily standard deviation of residuals times the square root of the number of trading days in that month. The inclusion of N_m transforms the daily return residuals into monthly residuals. This procedure can be can be seen in French et al. (1987) and Fu (2009).

Since the latent conditional volatility v_t^i cannot be directly observed, I use realized volatility: squared daily return residuals in month t obtained through the cross-sectional regression of equation (1) to measure the individual stock's idiosyncratic volatility for month t. Specifically,

$$IV_t^i \equiv \sqrt{\sum_{d=1}^{N_m} (\eta_{t,d}^i)^2} \tag{3}$$

When I refer to idiosyncratic volatility in this paper, I mean idiosyncratic volatility relative to the FF-3 model.

2.3 Time Series Properties of Realized Idiosyncratic Volatility

Table 1 presents the time-series properties of the realized idiosyncratic volatility (IV). I first compute the time-series statistics of idiosyncratic volatility for each firm and then summarize the mean statistics across about 22,000 firms. The mean of idiosyncratic volatility is 15.54% across stocks, and the mean standard deviation for IV is 9.21%. The skewness is 2.00, and kurtosis is 8.23, which suggests that the idiosyncratic volatility is positively skewed and fat-tailed. The autocorrelation for realized idiosyncratic volatility is 0.39 with 1-month lag, 0.31 with 2-month lag, 0.21 with 5-month lag, 0.12 with 10-month lag, and 0.12 with one-year lag. The autocorrelation of 0.39 with 1-month lag and 0.31 with 2-month lag suggests that shocks to idiosyncratic volatility are not very persistent within short horizons (a quarter). However, the autocorrelations decay slowly over longer periods, for example, over a year. The autocorrelation of realized idiosyncratic volatility with a lag period of 12 months is still more than 0.1. In comparison, the autocorrelation of an AR(1) process with first-order autocorrelation of 0.39 would predict that the autocorrelation with 12-month lag is less than 0.2 basis points [2].

[2] One basis point equals one hundredth of one percentage point.

[**Insert Table 1 about here**]

This pattern of a relatively quick initial decline in the autocorrelation function followed by a slower decay is sufficient evidence to dismiss using the simple ARMA model to model idiosyncratic volatility (for example, Barndorff-Nielsen and Shephard 2002). Corsi (2009) proposes that the aggregate stock return volatility could be captured by an additive cascade model of volatility defined over different time periods. The empirical evidence in this section suggests that the idiosyncratic volatility of stock returns could also be better captured by a process with components of different persistence. Therefore, I model the log of idiosyncratic volatility as the sum of a short-run and a long-run component. The short-run component is less persistent and has a large impact on the autocorrelations of idiosyncratic volatility over short horizons (a quarter). And the more persistent long-run component dominates the autocorrelations over longer horizons (a year or longer).

2.4 Decomposing Idiosyncratic Volatility

To decompose idiosyncratic volatilities into short-run and long-run components, I model the idiosyncratic volatility v_t^i as follows:

$$\begin{aligned} \textbf{Idiosyncratic Volatility}: & \log v_t^i = s_t^i + l_t^i \\ \textbf{Short-Run Component}: & s_{t+1}^i = \rho_s^i s_t^i + \sigma_s^i \epsilon_{s,t}^i \\ \textbf{Long-Run Component}: & l_{t+1}^i = \phi_i + \rho_l^i l_t^i + \sigma_l^i \epsilon_{l,t}^i \end{aligned} \tag{4}$$

I refer to this model as the short- and long-run (SL) model hereafter. In equation (4), the log-volatility is the sum of two components, s_t and l_t. Each component follows a first order autoregressive process AR(1) with its own rate of mean reversion. The short-run component s_t has a mean of zero, while the long-run component l_t contains a constant ϕ_i [3]. I normalize the mean reversion parameters such that $\rho_l > \rho_s$. This restriction identifies the model, as otherwise, the two components can be interchangable. Moreover, parameters σ_s and σ_l denote the volatility of shocks to the short-run and long-run components. Shocks to the short- and long-run components $\epsilon_{s,t}$ and $\epsilon_{l,t}$ are normal independently and identically distributed with zero expectation and unit variance.

For each firm, equation (4) is readily in a state space form, and the unobserved short-run and long-run components can be directly estimated via a Kalman filter. I consider $\hat{s}_t \equiv \mathbb{E}_{t-1}(s_t|y_1, y_2, ...y_{t-1})$ and $\hat{l}_t \equiv \mathbb{E}_{t-1}(l_t|y_1, y_2, ...y_{t-1})$ as the expectation for the short-run and long-run components at time t based on information available at time $t-1$. The smoothed estimates $\tilde{s_t} = \mathbb{E}(s_t|y_1, y_2, ...y_T)$ and $\tilde{l_t} = \mathbb{E}(l_t|y_1, y_2, ...y_T)$, which use all the sample information, may produce more precise estimates for the expectation of unobserved components s_t and l_t at each point in time. Therefore, full information set estimates are appropriate for asset pricing tests because of the gain in accuracy. However, in terms of evaluating trading strategies, incorporating future observations directly in forecasts may lead to using substantial information beyond what investors are aware of [4] . Thus, I report results using filtered estimates in the subsequent empirical analysis. Empirical results using smoothed estimates are statistically more significant but are nevertheless included in the Appendix.

Starting with Engle and Lee (1999), a number of studies find that two-component volatility models outperform one-component specifications in explaining equity market volatility. Adrian and Rosenberg (2008) consider a two components model for aggregate stock market volatility and find that the prices of risk are different for the short-run and long-run component. In addition, two-component volatility models perform well in the option pricing literature. For example,

[3]The constant term should be excluded from the short-run component mainly for two reasons. First, a constant term is capturing a very "persistent" part of idiosyncratic volatility. If a firm has a high constant ϕ_i, it tends to have high idiosyncratic volatility for long periods of time. This implication also plays an important role in interpreting the negative relationship between long-run idiosyncratic volatility and stock returns through real-option-based channels in Section 5.3. Second, the constant term in the long-run component makes the SL model to some extent comparable to the limiting case when the long-run component contains a unit root and does not have a mean.

[4]Filtered estimates may still use future information if parameters are estimated based on whole sample information.

Christoffersen et al. (2008) show that modeling stock return volatility with short- and long-run components perform well for option pricing. My paper differs from Adrian and Rosenberg (2008) both in the estimation method and the focus on idiosyncratic volatility. Adrian and Rosenberg (2008) estimate volatilities using a maximum-likelihood method on daily stock returns and then aggregate volatilities to monthly frequencies. I use a state-space model on realized idiosyncratic volatility to decompose realized idiosyncratic volatility into short-run and long-run components. And this method is convenient to extract filtered and smoothed estimates of conditional idiosyncratic volatility.

2.5 A Permanent and Transitory Special Case

In my empirical work, I also investigate a special case of equation (4) where the long-run component contains a unit root and the short-run component follows a white noise. I refer to this model as the permanent and transitory (PT) model.

$$\begin{aligned} \textbf{Idiosyncratic Volatility}: & \log v_t^i = s_t^i + l_t^i \\ \textbf{Short-run Component}: & s_{t+1}^i = \sigma_s^i \epsilon_{s,t}^i \\ \textbf{Long-run component}: & l_{t+1}^i = l_t^i + \sigma_l^i \epsilon_{l,t}^i \end{aligned} \tag{5}$$

Equation (5) may be viewed as a special case of (4) with the restriction that $\rho_s = 0$ and $\rho_l = 1$. The log-volatility is the sum of two components, s_t and l_t. The long-run component l_t follows a random walk. Thus changes to the long-run volatility could be permanent and are persistent over time. For each firm, equation (5) can also be estimated using a Kalman filter. Since the long-run component l_t follows a random walk, the one-step-ahead conditional expectation of the long-run component $\mathbb{E}_t(l_{t+1}|y_1, y_2, ..., y_t) = \mathbb{E}_t(l_t|y_1, y_2, ..., y_t)$ and that of the short-run component $\mathbb{E}_{t-1}(s_t|y_1, y_2, ..., y_{t-1})$ trivially equals zero. Therefore, for the permanent and transitory (PT) model, I only consider the expectation of the long-run component $\hat{l}_t = \mathbb{E}_{t-1}(l_t|y_1, y_2, ..., y_{t-1})$ and $\tilde{l}_t = \mathbb{E}(l_t|y_1, y_2, ..., y_T)$ and investigate their relationship with expected returns.

2.6 Parameter Estimates of the Idiosyncratic Volatility Model

In practice, the true conditional idiosyncratic volatility v_t cannot be directly observed. Consequently, the realized volatility IV_t is a proxy for the latent volatility subject to measurement errors. Because measurement errors are largely identically and independent distributed over time, it has little forecasting power for forming conditional expectations. In empirical studies, realized volatilities are usually treated as measuring latent volatilities without errors. (for example, Bollerslev and Zhou 2002; Chua et al. 2010). In this paper, I report results using this approach because it massively simplifies subsequent estimation and analysis. The results with identically and independently distributed measurement errors are quantitatively close and are reported in the Appendix.

Table 2 summarizes parameter estimates for the short- and long-run volatility (SL) model with equation (4) and the permanent transitory volatility (PT) model with equation (5). Both the SL and the PT model are estimated using the maximum likelihood method. For the SL model, the mean AR(1) parameter for the short-run component is -0.07, while the median is -0.003. The long-run component is more persistent, with a mean AR(1) coefficient of 0.79 and a median of 0.94. The mean volatility of shocks to the short-run component is 0.29 and the median is 0.31. For the long-run component, the mean volatility is 0.20 and the median is 0.15. Therefore, the short-run component doesn't persist long, but shocks to it are relatively bigger. It mostly fluctuates around the mean, zero. Despite that shocks to the long-run component tend to be smaller, the long-run component is relatively persistent, which means that the level of the long-run component can display substantial variations over time. In the Appendix, I also plot estimates of the short-run and long-run components of idiosyncratic volatility for a few randomly selected firms.

The permanent and transitory (PT) model can be viewed as a special case of the SL model with $\rho_s = 0$ and $\rho_l = 1$. Given that the median estimate of ρ_s is -0.003 and ρ_l is 0.94 from the SL model, the PT model can be a plausible model to capture the dynamics of idiosyncratic volatility. For the PT model, the only parameters to be estimated are the volatility of shocks to the short-run and long-run components. The mean volatility of shocks to the short-run component is 0.36, and the median is 0.34. As for the volatility of shocks to the long-run component, the mean is 0.14, and the median is 0.10. The magnitude of shocks is also largely similar to estimates from the SL

[**Insert Table 2 about here**]

model.

3 Portfolio Sorts of Idiosyncratic Volatility and Cross-Sectional Stock Returns

This section considers the performance of portfolios formed by different measures of idiosyncratic volatilities and asks whether exposures to different volatilities are systematically important for expected stock returns. To examine trading strategies based on idiosyncratic volatility, I consider the standard portfolio formation strategies following Jegadeesh and Titman (1993) with a holding period of $N = 1, 12, 24$ months. For strategies with multi-month holding periods, I average across the N subqunintiles formed at the beginning of month $t - s$, for $s = 0, 1, 2, 3, ...N - 1$, as the return for a given quintile.

3.1 Patterns in Average Returns for Idiosyncratic Volatility

I first consider value-weighted quintile portfolios formed every month by sorting stocks based on realized idiosyncratic volatility relative to the FF-3 model. Portfolios are formed every month based on realized volatility computed using daily data of the previous month. Panel A in Table 3 shows that the average return increases slightly from 0.96% per month to 1.05% going from the quintile 1 (low idiosyncratic volatility stocks) to quintile 3. Then portfolios' returns drop tremendously going to quintile 5. The portfolio with the highest idiosyncratic volatility (quintile 5) has a surprisingly low average return of 0.17% per month. The difference in returns between the highest and lowest portfolios is as large as −0.79% per month, which is statistically significant, with a robust t-statistic of −2.84. These numbers are similar to the findings of Ang et al. (2006), who find a return spread of −0.97% between quintile 5 and quintile 1 portfolios, with a significance of −2.97, in a July 1963–December 2000 sample.

The FF3-alpha for the quintile 5 portfolio is −1.21% per month, with a robust t-statistics of −7.15. Therefore, the difference in quintile portfolio returns can not be explained by our standard FF-3 model. The difference in average returns in Table 3 indicates a significant negative relationship between expected return and idiosyncratic volatility. Panel B in Table 3 also reports average portfolio returns with holding periods of $N = 12, 24$ months. For the holding period of $N = 12$ months, the average return decreases from 0.91% per month for the quintile 1 portfolio (low idiosyncratic volatility stocks) to 0.58% for the quintile 5 portfolio (high idiosyncratic volatility stocks). The difference in returns is −0.33%, with a robust t-statistics of −2.89. Similarly, when the holding period is $N = 24$ months, the average return of the quintile 1 portfolio is 0.89% per month, while it is 0.72% for the quintile 5 portfolio. The difference in returns is −0.17%, with a robust t-statistic of −2.12. Therefore, the negative relation between lagged idiosyncratic volatility and stocks returns holds for longer holding periods, suggesting that the cross-sectional relationship is persistent and expected returns have a persistent component.

[**Insert Table 3 about here**]

3.2 Portfolios Sorted by Short-Run and Long-Run Volatilities

Estimating the state-space model of (4) and (5) produces estimates of the conditional short-run and long-run volatilities at the monthly frequency. I form value-weighted quintile portfolios sorting by the filtered short-run volatility $\hat{s}_t$ and long-run volatility $\hat{l}_t$.

First, I consider the performance of portfolios sorted by the filtered estimates of the conditional long-run component with a holding period of one month. Table 4 reveals substantial spreads in average returns across quintile portfolios. For both the SL model and the PT model, the portfolio with the highest long-run volatilities earns particularly low average returns. The average return of the quintile 5 portfolio with the highest long-run idiosyncratic volatility is as low as 0.18% per month for the SL model and 0.25% for the PT model. The spread in average returns between the portfolios with the lowest and highest long-run volatility is -0.73% per month for the SL model and -0.68% for the PT model. The difference in returns also can not be explained by the FF-3 model. The highest long-run volatility portfolio has FF3-alpha of -1.25% for the SL model and -1.19% for the PT model. The alphas are statistically significant, with robust t-statistics of -6.06 and -5.72, respectively. Therefore, there is a strong negative relation between conditional long-run volatility and stock returns.

Next, Panels A and B in Table 6 report the portfolio performance of sorting on long-run idiosyncratic volatility for holding periods of $N = 12, 24$ on the filtered estimates of long-run idiosyncratic volatility. Statistical significance actually increases for longer holding periods. For the SL model, return spreads are -0.73%, -0.38%, and -0.22%, respectively, for $N = 1, 12, 24$. The corresponding t-statistics are -2.10, -2.67 and -2.22. Higher statistical significance suggests that conditional long-run idiosyncratic volatility play a crucial role behind the persistent negative relationship between idiosyncratic volatility and stock returns.

Last, Table 5 and Panel C in Table 6 report the performance of portfolios sorted by the filtered estimates of conditional short-run idiosyncratic volatility. The high minus low short-run volatility portfolio earns an average monthly return of 0.19%, with a t-statistic of 2.81, which indicates that there is a significant positive relation between conditional short-run idiosyncratic volatility and expected stock returns.. Extending the strategy for multiple holding periods of $N = 12$ and $N = 24$

reveals that this positive relationship is not persistent over time. Panel C in Table 6 reports that the average spread is 0.03 and the significance is 1.40 over 12-month holding periods. Moreover, the average spread over the 24-month holding period even becomes negative, namely -0.02, with a significance of -0.99. These results suggest that stock prices go up when the conditional idiosyncratic volatility increases. However, as the short-run volatility dies off, stock prices revert back over time. Given the low average persistence parameter ρ_s estimated, the speed of reversion in stock prices may be slow to some extent. Holding periods of 12 and 24 months seem to be relatively long. This suggests that there may be some degree of frictions (limited investor attention or trading frictions, for example) in slowing the reversion of stock prices.

[**Insert Tables 4, 5, and 6 about here**]

4 Cross-Sectional Regressions

My empirical analysis thus far is based on portfolio sorts. In this section, I investigate the cross-sectional relationship between average stock returns and estimated conditional idiosyncratic volatilities. I follow Fama and MacBeth (1973) by regressing cross-sectional stock returns on idiosyncratic volatilities and other firm characteristics on a monthly basis and calculate the time-series averages of the coefficients. My goal is to test whether the coefficient on idiosyncratic volatility in explaining cross-sectional stock returns is significantly different from zero.

Specifically, I run the following cross-sectional regressions each month for the SL and PT model:

$$R^e_{i,t+1} = \gamma_{l,t}\hat{l}_t + \gamma_{s,t}\hat{s}_t + \epsilon_{i,t+1} \tag{6}$$

$$R^e_{i,t+1} = \gamma_{l,t}\tilde{l}_t + \gamma_{s,t}\tilde{s}_t + \epsilon_{i,t+1} \tag{7}$$

where $r^e_{t+1,i}$ is stock i's excess return in month $t+1$ minus its Fama and French (1993) factor adjustments. Volatilities with tildes are smoothed estimates, and those with hat signs are filtered estimates.

Table 7 shows time-series averages of the coefficients from the month-by-month Fama–MacBeth (FM) regressions of the cross-section of stock returns on different measures of idiosyncratic volatility. The average coefficient on variables used to explain expected returns provides standard FM tests for determining which variables on average have explanatory power during the July 1963 to December 2017 period [5].

The average coefficient on the log of realized idiosyncratic volatility (IV) is -0.52, with a t-statistic of -5.66. The finding confirms the negative relationship between idiosyncratic volatility and expected return found by Ang et al. (2006).

[**Insert Table 7 about here**]

Subsequently, I include measures of conditional short-run and long-run idiosyncratic volatility into FM regressions. Regressions using the SL model are reported in Table 7. The regression results

[5] All results with smoothed estimates are reported in the appendix.

indicate that there exists a negative (positive) relationship between conditional long-run (short-run) volatility and expected returns. The average coefficient is -0.50 (2.41), with a significant t-statistic of -4.01 (6.10) for the filter long-run (short-run) component. The statistical significance is also higher for the smoothed estimates. These FM regression results lend support to the finding in Section 3.2 with stocks sorted by $\hat{s}_t$ and $\hat{l}_t$. It is also useful to explain the finding using the PT model to measure conditional long-run idiosyncratic volatility. Table 8 shows that the average coefficient on the filtered estimates of conditional long-run volatility $\hat{l}_t$ is -0.49, with a t-statistic of -4.11.

4.1 Additional Robustness Check

As robustness checks, I include two additional controls in the Fama–MacBeth regressions — return reversals and unexpected idiosyncratic volatility — in this section. The findings in the previous section remain robust when controlling for these two channels.

4.1.1 Controlling for Return Reversals

Stock returns display short-term reversals (Jegadeesh 1990; Lehmann 1990). Return reversal describes the phenomenon that if a stock's previous-month return is too high (low), it will tend to reverse the following month and earn a low (high) return. Following Huang et al. (2010), I use the returns of individual stocks in the prior month to control for return reversals. Therefore, equation (7) is modified to allow for the previous month's stock return:

$$r^i_{t,d} = \gamma_{l,t}\hat{l}_t + \gamma_{s,t}\hat{s}_t + \beta_{r,t-1}r^i_{t-1} + v^i_t\epsilon^i_{t,d} \tag{8}$$

Without the previous month's stock return r^i_{t-1}, the relationship between idiosyncratic risk and expected stock returns may be negatively biased because the coefficient incorporates part of the return reversal that should have been captured by the stock return of the previous month. Including return reversals in the FM regression, Table 7 shows that the coefficient on the log of realized volatility is reduced to -0.42 with a t-statistic of -4.55. This finding is consistent with the finding in Huang et al.'s (2010) that part of the finding by Ang et al. (2006) can be explained by return reversals. And the coefficient on the lagged month return is statistically significant, with a statistic of -10.59.

However, accounting for return reversals doesn't quite reduce the coefficients of the conditional short-run and long-run components. Some of the coefficients become even more significant after the lagged month return control is added. Still, the coefficient on lagged month return is significant for both the SL and the PT model, with t-statistics of -11.36 and -12.23. Therefore, return reversals are not the key driver of the relationship between short-run and long-run conditional idiosyncratic volatility and expected returns.

4.1.2 Controlling for Unexpected Idiosyncratic Volatility

In the spirit of the argument of French et al. (1987), the relationship between expected idiosyncratic volatility and expected returns in the above regression may be clouded by the relationship between unexpected idiosyncratic volatility and unexpected stock returns. To control for this effect, I add unexpected idiosyncratic volatility to the FM regression. I define the unexpected idiosyncratic volatility as

$$\mu_t = \log v_t - \hat{s}_t - \hat{l}_t$$

where $\log v_t$ is the log of realized volatility at time t, and $\hat{s}_t$ and $\hat{l}_t$ are filtered volatility estimates made at time $t-1$ for the short-run and long-run components at time t. The average of first-order autocorrelations of μ_t is 0.003, suggesting μ_t has no time-series predictability. The average of correlations between s_t and μ_t is 0.07, and it is -0.06 between s_t and l_t. These statistics justify μ_t as being unexpected. When the unexpected idiosyncratic volatility is added to the regression, cross-sectional relationships between conditional short-run and long-run idiosyncratic volatility and expected stock returns remain robust.

Table 7 reports results for the SL model. The coefficient on the filtered short-run (long-run) idiosyncratic volatility is 3.52 (-0.34) with a robust t-statistic of 8.92 (-2.66), which further supports that there is a strong negative (positive) relationship between the conditional long-run (short-run) component and average stock returns. The coefficient on the unexpected idiosyncratic volatility is also significant. For the filtered idiosyncratic volatility estimates group, the coefficient of unexpected idiosyncratic volatility is 4.81, with a high t-statistic of 22.61. The positive relationship between unexpected idiosyncratic volatility and stock returns is consistent with the positive contemporaneous relationship between stock returns and firm-level idiosyncratic volatility found by Duffee (1995) and Grullon et al. (2012) [6]. Table 8 reports similar results of adding unexpected idiosyncratic volatility to the PT model.

5 The Cross-Sectional Relationships Between Short-Run and Long-Run Idiosyncratic Volatility and Expected Stock Returns

In this section, I examine the risk exposures of portfolios sorted by short-run and long-run idiosyncratic volatility. Then I attempt to interpret these results through possible channels through which idiosyncratic volatility is related to stock returns.

5.1 Investigating Risk Exposures

To investigate whether the cross-sectional relationship between short- or long-run idiosyncratic volatility and stock returns can be explained by risk exposures, I examine return differences between portfolios sorted by short-run and long-run idiosyncratic volatility. If these return differences are driven by risk, they should comove with systematic risk factors. If no significant correlations are found, the relation could then be non-risk-based and driven by forces such as market frictions.

To this end, I compute the correlations between the returns of the high minus low (5-1) portfolio sorted by conditional short-run idiosyncratic volatility, long-run idiosyncratic volatility, and lagged realized volatility, and the five factors of Fama and French (2015) (FF-5). Based on the evidence of Novy-Marx (2013) and Titman et al. (2004), profitability factor RMW and investment factor

[6] The convexity-based real-option explanation in Section 5.3 may be useful to explain the positive sign on unexpected idiosyncratic volatility. The average of correlations between the conditional long-run volatility at $t+1$: $\hat{l}_{t+1}$ and unexpected idiosyncratic volatility μ_t is about 0.4. This suggests that a positive unexpected change in μ_t is associated with an upward revision in the conditional long-run volatility $\hat{l}_{t+1}$. When the long-run idiosyncratic volatility $\hat{l}_{t+1}$ is expected to increase, the stock price at t may rise as growth options become more valuable. The expected return at $t+1$ falls as the exposure of growth options to systematic risk factors decreases, though.

CMA are added in addition to the three factors of Fama and French (1993) [7]. As for the short-run and long-run components, I use the filtered estimates here, which use information at time $t-1$ to predict the conditional volatility at time t.

Table 9 reports the correlations between portfolio spreads sorted by idiosyncratic volatility and the five factors. It is found that the return spread sorted by the conditional short-run component, denoted by IVFS, is not correlated with the spread sorted by the conditional long-run idiosyncratic volatility IVFL, realized idiosyncratic volatility IVFR, or the five factors of Fama and French (2015). The correlation is -0.04 with the IVFL portfolio, 0.02 with the excess market return, 0.06 with the size factor SMB, 0.04 with the value factor HML, 0.04 with the RMW factor, and 0.04 with the CMA factor. The lack of correlations with systematic risk factors further suggests that the cross-sectional relationship between the conditional short-run component and stock returns is not likely driven by risk.

In the meantime, the return spread sorted by the conditional long-run component, denoted as IVFL, is strongly correlated with the return spread sorted by realized idiosyncratic volatility, denoted as IVFR, with a correlation of 0.95. Therefore, the cross-sectional relationship between realized idiosyncratic volatility and stock returns found by Ang et al. (2006) is mostly captured by the long-run idiosyncratic volatility. Besides, the return spread IVFL correlates with the book-to-market factor HML, profitability factor RMW, and investment factor CMA with a negative sign. The correlation with the profitability factor is especially strong, with a coefficient of -0.62. Given that these factors may earn positive risk premiums, low exposure to them could help explain why stocks with high conditional long-run idiosyncratic volatility earn low returns.

Table 10 systematically examines how these five factors are useful to explain the portfolio returns sorted by the realized idiosyncratic volatility beyond the FF-3 model. The test assets used here are the 25 portfolios formed monthly on size and realized idiosyncratic volatility (Size-IV), provided on Kenneth French's website [8]. Similar to the findings of Fama and French (2016), the FF-5 model

[7] The question investigated here is not related to momentum. Including the momentum factor of Carhart (1997) doesn't impact the main results of this paper and the results are available upon request.

[8] According to information on Kenneth French's website,, the portfolios, which are constructed monthly, are the intersections of five portfolios formed on size (market equity, ME) and five portfolios formed on the variance of the residuals from the FF-3 model.

[**Insert Table 9 about here**]

provides a better description of average returns on these 25 portfolios. Table 12 reports Gibbons et al.'s (1989) GRS statistic, which is reduced from 6.76 for the three-factor model to 5.56 for the five-factor model. The average absolute alpha is also reduced significantly from 0.24 to 0.13. In particular, Table 10 shows that stocks with high idiosyncratic volatility have significant negative exposure to the profitability factor RMW.

While the profitability factor is useful to explain the negative relationship between idiosyncratic volatility and the cross-section of stock returns, it does not fully capture it. Strong exposure to RMW still misses part of the low average returns of high realized idiosyncratic volatility small stocks. Therefore, I investigate whether there is an additional factor structure behind the negative relationship between idiosyncratic volatility and the cross-section of stock returns. In particular, I investigate whether IVFL is a useful factor in explaining the cross-sectional relationship between idiosyncratic volatility and stock returns. This is testing whether there is a "slope" structure between portfolios sorted by idiosyncratic volatility.

Tables 11 and 12 report the results of adding the IVFL factor to the FF-5 model. As Table 12 shows, adding the IVFL factor does not lead to substantial gains beyond the FF-5 model. The GRS statistic is reduced from 5.55 to 5.15, and the average absolute alpha decreases from 0.13 to 0.11. However, Table 11 also shows that after the IVFL factor is added, coefficients on the RMW factor become almost insignificant. This pattern suggests that the information of RMW is largely captured by the IVFL factor. Therefore, it may be worthwhile for future research to investigate additional risk factor structures behind the negative relationship between conditional long-run idiosyncratic volatility and stock returns.

The empirical analysis in this section complements the paper by Guo and Savickas (2010). Guo and Savickas (2010) demonstrate that the difference in returns between low and high realized idiosyncratic volatility stocks is a priced factor in the cross-section of stock returns. And this factor is correlated with the value factor HML defined by Fama and French (1993). However, they haven't examined risk exposures to the FF-5 model. This paper examines whether return differences between low and high conditional short-run and long-run volatility are priced factors and whether they are related to the five factors of Fama and French (2015).

5.2 Why is Short-Run Idiosyncratic Volatility Related to Stock Returns?

Traditional asset pricing theories assuming full information, frictionless and complete markets predict no relationship between idiosyncratic volatility and expected returns when agents are rational. In reality, investors may not hold perfectly diversified portfolios. Various theories assuming under-diversification predict that idiosyncratic risk is positively related to expected stock returns, such as, informational frictions (Merton, 1987) and transaction costs (Hirshleifer, 1988). However, frictions that prevent investors from adjusting portfolios to perfectly diversify risk are more significant in short horizons. Moreover, the difficulty of achieving perfect diversification is more evident in short horizons when shocks to idiosyncratic volatility have a relatively large and short-lived component. In the SL model, the median estimates of the volatility of shocks to the short-run component θ_s is more than 50% larger than that of the long-run component θ_l.

For returns measured over long horizons, frictions such as transaction costs and limited attention tend to play restricted roles, which is consistent with that reported in Section 3.2. The return spread sorted by the conditional short-run idiosyncratic volatility is found not to hold for long holding periods of 12 and 24 months. Therefore, the positive relationship between conditional short-run idiosyncratic volatility and cross-sectional stock returns might arise because investors face frictions and hold under-diversified portfolios.

5.3 Why is Long-Run Idiosyncratic Volatility Related to Stock Returns?

Ang et al. (2006) find that stocks with high realized idiosyncratic volatility in one month earn extremely low average returns in the next month. One reason for this novel finding is that earlier studies do not sort stocks or examine idiosyncratic volatility at the stock level. In Section 5.1, I present empirical evidence that the return spreads between the lowest and highest quintile portfolio sorted by the conditional long-run idiosyncratic volatility and lagged realized idiosyncratic volatility are strongly correlated with a coefficient of 0.95. Additionally, Section 3.2 shows that the magnitude of the return spreads is also quantitatively close, with -0.79% for the realized volatility and -0.73% for the conditional long-run idiosyncratic volatility. This finding suggests that long-run idiosyncratic volatility plays a crucial role in the negative relationship between realized idiosyncratic volatility and stock returns.

Furthermore, as shown in Section 3.2 and 5.1, the cross-sectional relationship between long-run idiosyncratic volatility and stock returns persists over multi-period holding returns. And stocks with high long-run idiosyncratic volatility may be less exposed to systematic factors, especially the profitability factor RMW. All this empirical evidence lends support to risk-based explanations of the negative relationship between idiosyncratic volatility and cross-sectional stock returns. Moreover, risk-based mechanisms should explain why such relationship is limited to the long-run component.

There are several risk-based explanations for the negative relationship between idiosyncratic volatility and stock returns in the literature. For this type of explanation, idiosyncratic volatility can serve as a proxy for either exposure to systematic risk factors or sensitivity to fluctuations in changing investment opportunities as in Merton (1973).

Babenko et al. (2016) view firms as portfolios of separate systematic and idiosyncratic divisions and rely on the additivity of systematic and idiosyncratic cash flow shocks in the valuation of firms. Hence, favorable idiosyncratic shocks decreases the importance of systematic cash flows, leading to lower risk premia and higher idiosyncratic stock return volatility. Similarly, Chen et al. (ance) study a risk-shifting problem of equity householders who take on more investments with high idiosyncratic risk when firms are in distress and when the aggregate economy is in a bad state. Thus, the negative covariance between the equity beta and market risk premium in the conditional CAPM may explain the negative excess returns and negative CAPM alphas in the high-idiosyncratic-volatility firms.

Recent studies such as Cao et al. (2008) and Grullon et al. (2012), find that firms with high idiosyncratic volatility usually possess abundant growth options. Real options models, following Berk et al. (1999) and Carlson et al. (2004), establish links between expected returns and the riskiness of assets in place and growth options. The firm's systematic risk could crucially depend upon the risk exposure of its growth options, when such options' value consist of a large proportion of the firm's value. Bhamra and Shim (2017) introduce stochastic cash flow risks into an equity evaluation model with growth options to explain the negative relationship between idiosyncratic volatility and expected stock returns. This real-option-based mechanism highlights the importance of long-run idiosyncratic volatility in explaining the negative relationship between idiosyncratic volatility and stock returns. When idiosyncratic volatility increases, the value of growth options could rise due to convexity. Moreover, such a rise in the value of growth options could be significant if the increase in idiosyncratic volatility is occurs over long horizons and there is a possibility of delaying investment. Short-run variations in idiosyncratic volatility that level off quickly over time may have a very limited impact on option values.

In the meantime, growth options' sensitivity to systematic risk factors could decrease because the relative magnitude of such options' value that is related to systematic risk falls. This channel drives down the expected return when idiosyncratic volatility is higher. Therefore, long-run idiosyncratic volatility serves as a proxy for exposure to systematic risk factors. A simple model similar to that of Bhamra and Shim (2017) is also provided in the Appendix to shed light on the real-option-based channel to explain the negative relationship between idiosyncratic volatility and stock returns.

Guo and Savickas (2008, 2010), motivated by the reasoning that idiosyncratic volatility could be related to growth options and investment opportunities, show that CAPM, FF-3 model, and other asset pricing models may suffer from omitted variable bias because they do not include a measure of the set of investment opportunities proposed in Merton's (1973) ICAPM. As a result, their pricing relationships assign too high a price of risk to the changes in aggregate investment opportunities, which imparts a negative expected return on idiosyncratic volatility. Along with this argument, if long-run idiosyncratic volatility is a good proxy for changing investment opportunities, this mechanism may also explain the negative relationship between long-run idiosyncratic volatility and cross-sectional stock returns.

As for the non-risk-based explanations, the list of them could include lottery preferences (Bali et al., 2011), limits to arbitrage (Stambaugh et al., 2015), and so forth. In a recent paper, Stambaugh et al. (2015) argue that costly arbitrage leads to the pricing of idiosyncratic risk, but the cost is higher for overpriced stocks than for underpriced ones. Due to the fact that the negative relation among overpriced stocks is stronger, especially for stocks less easily shorted, the overall relation between idiosyncratic volatility and stock returns is negative. Since mispricing tends to be corrected over the long-run, it is unclear whether this explanation could generate a persistent negative relation between idiosyncratic volatility and stock returns. It is thus worth investigating whether their findings hold over longer return horizons. Furthermore, non-risk-based explanations may be challenged to reconcile the finding in this paper that the relationship between idiosyncratic volatility and cross-sectional stock return is limited to the long-run component.

Therefore, there are two real-option-related mechanisms that may explain the negative relation between long-run idiosyncratic volatility and stock returns. One possible avenue for future research is to rigorously test these real-option-based and other mechanisms in explaining the negative relationship between long-run idiosyncratic volatility and cross-sectional stock returns.

6 Conclusion

The paper develops and estimates a model that better captures the dynamics of idiosyncratic volatility. I decompose the volatility of idiosyncratic stock returns into short-run and long-run components and find that there is a significant negative (positive) relationship between conditional long-run (short-run) idiosyncratic volatility and expected stock returns. And the negative relationship between lagged realized volatility and expected returns is captured by the long-run component. These results highlight that idiosyncratic variations over short and long horizons have important implications for cross-sectional stock returns.

Further empirical tests suggest the positive relationship between short-run idiosyncratic volatility and stock returns is not risk-driven, while the negative relationship between long-run idiosyncratic volatility and stock returns may be driven by risk. Moreover, the finding that the negative relationship between realized idiosyncratic volatility and stock returns is limited to the long-term component suggests two types of real-option-based mechanisms to explain the negative relationship between long-run idiosyncratic volatility and stock returns.

One explanation is that stocks with high long-run idiosyncratic volatility are less exposed to systematic risk factors. The systematic risk of a firm is determined by the risk of its assets in place and growth options. When idiosyncratic volatility increases, growth options rise in value because of convexity. And the exposure of growth options to systematic risk factors could fall due to the decrease in the relative magnitude of option values related to systematic risk factors. The other explanation is related to Merton's (1973) ICAPM. Long-run idiosyncratic volatility might be proxying for sensitivity to fluctuations in changing investment opportunities.

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Tables and Figures

Table 1: Time Series Properties of Idiosyncratic Volatility

Panel A: Some Summary Statistics of Idiosyncratic Volatility						
Mean	Std.Dev.	Skewness	Kurtosis			
15.54	9.21	2.00	8.23			
Panel B: Autocorrelations of Idiosyncratic Volatility						
ACF(1)	ACF(2)	ACF(3)	ACF(4)	ACF(5)	ACF(10)	ACF(12)
0.39	0.31	0.28	0.23	0.21	0.12	0.12

This table summarizes the time-series statistics for idiosyncratic volatility. I first compute the statistics for each stock and then average the statistics across all stocks. The sample period is July 1963 to December 2017. The ACF stands for estimated autocorrelations at different lags. The unit of the mean and standard deviation is percentage points.

Table 2: Parameter Estimates for Idiosyncratic Volatility Model

Panel A: The Short- and Long-Run Volatility (SL) Model				
Variables	ρ_s	ρ_l	σ_s	σ_l
Mean	-0.07	0.79	0.29	0.20
Median	-0.003	0.94	0.31	0.15
Panel B: The Permanent and Transitory Volatility Model				
Variables	σ_s	σ_l		
Mean	0.36	0.14		
Median	0.34	0.10		

This table summarizes the properties of parameter estimates for the short- and long-run idiosyncratic volatility processes. I first compute parameter estimates for each stock and then construct the mean and median statistics across all stocks. The sample period is July 1963 to December 2017.

Table 3: Portfolios Sorted by Idiosyncratic Volatility

Panel A: Ranking on Realized Idiosyncratic Volatility

Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.96	3.65	43.8%	0.11
				[3.14]
2	0.96	4.59	31.3%	0.00
				[0.08]
3	1.05	5.69	15.3%	-0.01
				[-0.17]
4	0.76	7.03	7.1%	-0.37
				[-3.82]
5 (high)	0.17	8.38	2.5%	-1.10
				[-7.40]
$5-1$	-0.79	6.66		-1.21
	[-2.84]			[-7.15]

Panel B: Ranking on Realized Idiosyncratic Volatility with Multiple Holding Periods

Period	1 Low	2	3	4	5 High	$5-1$
$N=1$	0.96	0.96	1.05	0.76	0.17	-0.79
						[-2.84]
$N=12$	0.91	0.95	0.97	0.89	0.58	-0.33
						[-2.89]
$N=24$	0.89	0.94	0.97	0.94	0.72	-0.17
						[-2.12]

I form value-weighted quintile portfolios every month by sorting stocks based on idiosyncratic volatility relative to the FF-3 model. Portfolios are formed every month, based on volatility computed using daily data of the previous month. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. The column $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column reports Jensen's alpha with respect to the FF-3 model. Robust Newey and West (1987) t-statistics with up to one lag are reported in square brackets. The sample period is July 1963 to December 2017.

Table 4: Portfolios Sorted by the Filtered Estimates of Conditional Long-Run Volatility

Panel A: Short and Long-Run Volatility (SL) Model				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.92	3.68	47.2%	0.09
				[2.50]
2	1.00	4.77	32.0%	0.00
				[0.06]
3	1.05	6.21	13.7%	-0.01
				[-0.1]
4	0.86	7.98	5.6%	-0.31
				[-2.73]
5 (high)	0.18	9.88	1.5%	-1.15
				[-6.26]
$5-1$	-0.73	8.26		-1.25
	[-2.10]			[-6.06]

Panel B: Permanent and Transitory Volatility (PT) Model				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.93	3.67	46.8%	0.10
				[2.90]
2	1.00	4.75	32.0%	-0.00
				[-0.09]
3	1.05	6.14	13.9%	-0.01
				[-0.17]
4	0.84	7.91	5.7%	-0.33
				[-2.88]
5 (high)	0.25	9.84	1.6%	-1.09
				[-5.79]
$5-1$	-0.68	8.23		-1.19
	[-1.94]			[-5.72]

Portfolios are formed every month based on the filtered estimates of conditional short-run idiosyncratic volatility $\hat{l}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. The column $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column reports Jensen's alpha with respect to the FF-3 model. Robust Newey and West (1987) t-statistics with up to one lag are reported in square brackets. The sample period is July 1963 to December 2017.

Table 5: Portfolios Sorted by Filtered Estimates of Expected Short-Run Volatility

Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.77	4.63	12.3%	-0.16
				[-2.84]
2	0.89	4.44	23.4%	-0.02
				[-0.44]
3	0.98	4.42	28.2%	0.07
				[2.31]
4	0.91	4.45	23.4%	0.01
				[0.39]
5 (high)	0.96	4.66	12.7%	0.01
				[0.25]
$5-1$	0.19	1.74		0.17
	[2.81]			[2.40]

Portfolios are formed every month based on the filtered conditional short-run idiosyncratic volatility $\hat{s}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. The column $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column reports Jensen's alpha with respect to the FF-3 model. Robust Newey and West (1987) t-statistics with up to one lag are reported in square brackets. The sample period is July 1963 to December 2017.

Table 6: Portfolios Sorted by Idiosyncratic Volatility with Multiple Holding Periods

Panel A: Ranking on Conditional Long-Run Idiosyncratic Volatility: PT						
Period	1 Low	2	3	4	5 High	$5-1$
$N=1$	0.93	1.00	1.05	0.84	0.25	-0.68
						[-1.94]
$N=12$	0.89	0.99	1.01	0.94	0.55	-0.34
						[-2.46]
$N=24$	0.87	0.98	1.03	0.99	0.69	-0.19
						[-1.96]
Panel B: Ranking on Conditional Long-Run Idiosyncratic Volatility: SL						
Period	1 Low	2	3	4	5 High	$5-1$
$N=1$	0.92	1.00	1.05	0.86	0.18	-0.73
						[-2.10]
$N=12$	0.89	0.99	1.02	0.94	0.51	-0.38
						[-2.67]
$N=24$	0.87	0.98	1.03	0.99	0.66	-0.22
						[-2.22]
Panel C: Ranking on Conditional Short-Run Idiosyncratic Volatility: SL						
Period	1 Low	2	3	4	5 High	$5-1$
$N=1$	0.77	0.89	0.98	0.91	0.96	0.19
						[2.81]
$N=12$	0.87	0.90	0.94	0.89	0.90	0.03
						[1.40]
$N=24$	0.90	0.89	0.93	0.89	0.88	-0.02
						[-0.99]

Panel A reports the performance of portfolios sorted by conditional long-run idiosyncratic volatility for the Permanent and Transitory (PT) model. Panel B and Panel C report the performance of portfolio sorted by the conditional short-run and long-run idiosyncratic volatility for the short- and long-run (SL) model. I form value-weighted quintile portfolios every month by sorting stocks based on idiosyncratic volatility relative to the FF-3 model. The holding period is 1 month, 12 months or 24 months. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess returns. The column $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column reports Jensen's alpha with respect to the FF-3 model. Robust Newey and West (1987) t-statistics with up to one lag are reported in square brackets. The sample period is July 1963 to December 2017.

Table 7: Relationship between Idiosyncratic Volatility and Expected Returns: SL Model

Short and Long-Run Volatility (SL) Model				
$\log v_t$	$\hat{s}_t$	$\hat{l}_t$	$Ret(-1)$	μ_t
-0.52				
[-5.66]				
	2.41	-0.50		
	[6.10]	[-4.01]		
-0.42			-4.82	
[-4.46]			[-10.67]	
	2.70	-0.48	-5.16	
	[6.81]	[-3.74]	[-11.43]	
	3.52	-0.34	-4.34	4.81
	[8.93]	[-2.66]	[-10.19]	[22.61]

The average coefficient is the time-series average of monthly regression coefficients from July 1963 to December 2017, and the t-statistic is the average coefficient divided by its time-series standard error. The t-statistic is reported in brackets.

Table 8: Relationship between Idiosyncratic Volatility and Expected Returns: PT Model

Permanent and Transitory Volatility (PT) Model			
$\log v_t$	$\hat{l}_t$	$Ret(-1)$	μ_t
-0.52			
[-5.73]			
	-0.49		
	[-4.11]		
-0.42		-4.79	
[-4.55]		[-10.59]	
	-0.48	-5.07	
	[-3.88]	[-11.21]	
	-0.26		4.88
	[-2.12]		[23.34]
	-0.26	-4.19	4.75
	[-2.08]	[-9.72]	[22.90]

The average coefficient is the time-series average of monthly regression coefficients from July 1963 to December 2017, and the t-statistic is the average coefficient divided by its time-series standard error. The t-statistic is reported in brackets.

Table 9: Correlations of Return Spreads with the Five Factors of Fama and French (2015)

	IVFS	IVFL	IVFR	$R_M - R_F$	SMB	HML	RMW	CMA
IVFS	1.00	-0.04	-0.00	0.02	0.06	0.04	0.04	0.04
IVFL	-0.04	1.00	0.95	0.52	0.68	-0.33	-0.62	-0.37
IVFR	-0.00	0.95	1.00	0.50	0.66	-0.31	-0.58	-0.36
$R_M - R_F$	0.02	0.52	0.50	1.00	0.28	-0.26	-0.23	-0.38
SMB	0.06	0.68	0.66	0.28	1.00	-0.07	-0.35	-0.10
HML	0.04	-0.33	-0.31	-0.26	-0.07	1.00	0.06	0.70
RMW	0.04	-0.62	-0.58	-0.23	-0.35	0.06	1.00	-0.04
CMA	0.04	-0.37	-0.36	-0.38	-0.10	0.70	-0.04	1.00

The table reports pairwise correlations between return spreads of the high minus low portfolio and the five factors of Fama and French (2015). The variable IVFS denotes the return spread of sorting stocks by the conditional short-run idiosyncratic volatility, IVFL denotes the return spread sorted by the conditional long-run idiosyncratic volatility, and IVFR, the return spread sorted by lagged realized idiosyncratic volatility. $R_M - R_F$ is the excess market return, SMB is the size factor, HML is the book-to-market factor, RMW is the profitability factor, and CMA is the investment factor.

Table 10: Regression Tests for the FF-5 Model

IVOL→	Low	2	3	4	High	Low	2	3	4	High
Panel A: Three-factor $R_{i,t} - R_{F,t} = a_i + b_i(R_{M,t} - R_{F,t}) + s_i SMB_t + h_i HML_i + \epsilon_{i,t}$										
			a					t		
Small	0.38	0.33	0.10	-0.19	-1.24	5.30	4.55	1.41	-2.19	-8.36
2	0.28	0.24	0.19	0.03	-0.73	4.25	3.46	2.61	0.38	-7.59
3	0.17	0.20	0.12	0.09	-0.47	2.56	2.86	1.72	1.10	-4.93
4	0.19	0.15	0.07	0.04	-0.33	2.44	1.96	0.96	0.56	-3.26
Big	0.09	0.10	0.02	-0.06	-0.09	1.57	1.88	0.35	-1.00	-1.00
Panel B: Five-factor $R_{i,t} - R_{F,t} = a_i + b_i(R_{M,t} - R_{F,t}) + s_i SMB_t + h_i HML_i + r_i RMW_i + c_i CMA_i \epsilon_{i,t}$										
			a					$t(a)$		
Small	0.27	0.19	0.05	-0.10	-0.87	3.78	2.56	0.53	-0.90	-5.56
2	0.14	0.06	0.02	-0.08	-0.47	2.32	0.98	0.36	-1.12	-5.47
3	0.03	0.04	-0.05	-0.05	-0.25	0.41	0.61	-0.74	-0.63	-2.84
4	0.04	-0.03	-0.09	-0.07	-0.10	0.47	-0.36	-1.29	-0.91	-1.06
Big	0.01	-0.04	-0.09	-0.05	0.12	0.14	-0.74	-1.75	-0.86	1.45
			b					$t(b)$		
Small	0.71	0.98	1.09	1.15	1.13	35.36	45.24	38.38	29.88	21.22
2	0.78	1.01	1.12	1.24	1.27	45.88	55.58	54.33	49.29	34.55
3	0.79	0.99	1.10	1.19	1.25	46.95	45.34	51.49	47.21	39.80
4	0.82	0.99	1.13	1.20	1.26	36.27	42.61	52.37	48.92	42.61
Big	0.83	0.97	1.05	1.11	1.18	56.75	66.03	75.18	69.87	46.26
			s					$t(s)$		
Small	0.67	0.93	1.04	1.19	1.35	25.22	26.10	22.92	18.90	16.48
2	0.56	0.75	0.83	0.94	1.12	23.03	27.37	24.24	22.13	24.05
3	0.31	0.47	0.57	0.69	0.82	12.59	13.91	16.42	16.61	19.64
4	0.10	0.18	0.23	0.32	0.51	3.16	4.98	6.15	8.29	13.93
Big	-0.28	-0.25	-0.17	-0.12	0.03	-13.11	-12.65	-7.28	-4.84	0.75
			h					$t(h)$		
Small	0.40	0.35	0.34	0.28	0.22	9.72	6.79	4.50	2.82	1.93
2	0.34	0.33	0.26	0.18	-0.07	10.55	7.18	4.93	2.84	-0.88
3	0.33	0.35	0.28	0.17	-0.16	8.78	6.65	5.24	2.72	-2.51
4	0.30	0.26	0.21	0.12	-0.18	5.63	4.57	3.92	2.28	-3.25
Big	0.12	-0.04	0.05	0.06	-0.19	3.91	-1.55	1.38	1.68	-3.28
			r					$t(r)$		
Small	0.28	0.35	0.17	-0.15	-0.83	4.94	5.22	1.97	-1.31	-6.32
2	0.32	0.42	0.41	0.32	-0.53	8.92	7.80	6.37	4.17	-7.79
3	0.31	0.41	0.43	0.35	-0.45	7.58	6.54	7.29	5.43	-7.99
4	0.31	0.40	0.37	0.27	-0.51	6.51	6.62	6.27	4.73	-8.81
Big	0.18	0.26	0.25	0.00	-0.42	4.59	7.23	5.55	0.11	-7.72
			c					$t(c)$		
Small	0.07	0.08	-0.02	-0.22	-0.42	1.24	1.30	-0.24	-1.71	-2.05
2	0.14	0.15	0.10	-0.03	-0.39	3.07	3.03	1.86	-0.49	-4.11
3	0.19	0.05	0.10	0.02	-0.32	3.64	0.95	1.93	0.32	-3.67
4	0.21	0.15	0.12	0.07	-0.25	2.96	2.51	1.95	0.99	-2.79
Big	0.08	0.22	0.12	-0.05	-0.34	1.54	4.11	2.47	-1.03	-4.12

The LHS variables in each set of 25 regressions are monthly excess returns on the 25 Size-IV portfolios. The RHS variables are excess market return $R_M - R_F$, the size factor SMB, the value factor HML, the profitability factor RMW, and the investment factor CMA. Panel A shows intercepts from the three-factor model and Panel B shows intercepts and slopes and the five-factor model. The sample period is July 1963 to December 2017.

Table 11: Regression Tests for the FF-5-plus-IVFL Model

Five-factor plus IVFL: $R_{i,t} - R_{F,t} = a_i + b_i(R_{M,t} - R_{F,t}) + s_i SMB_t + h_i HML_i + r_i RMW_i + c_i CMA_i + v_i IVFL_i \epsilon_{i,t}$

			a					$t(a)$		
Small	0.23	0.17	0.09	0.02	-0.61	3.25	2.33	1.00	0.15	-4.19
2	0.07	0.00	-0.01	-0.07	-0.33	1.22	0.07	-0.16	-0.94	-3.90
3	-0.05	-0.01	-0.07	-0.05	-0.12	-0.78	-0.14	-1.02	-0.71	-1.47
4	-0.04	-0.08	-0.11	-0.07	0.04	-0.56	-1.13	-1.65	-0.92	0.43
Big	-0.05	-0.07	-0.11	-0.04	0.26	-0.92	-1.48	-1.95	-0.64	3.40
Small	0.74	0.99	1.06	1.07	0.94	36.29	48.77	41.31	32.23	19.59
2	0.83	1.05	1.14	1.23	1.16	53.42	64.33	62.59	53.95	47.78
3	0.84	1.02	1.11	1.20	1.16	53.67	55.37	52.28	47.62	46.81
4	0.88	1.03	1.15	1.20	1.16	39.94	50.03	56.69	48.60	45.35
Big	0.88	1.00	1.06	1.10	1.08	59.36	64.61	74.01	70.45	53.65
			s					$t(s)$		
Small	0.76	0.96	0.94	0.92	0.71	14.81	15.48	13.57	11.04	8.66
2	0.73	0.89	0.92	0.91	0.75	17.62	17.66	15.75	14.74	12.47
3	0.49	0.60	0.62	0.71	0.50	11.17	9.52	11.25	12.78	10.44
4	0.29	0.31	0.29	0.33	0.18	5.75	5.33	5.39	6.33	3.01
Big	-0.13	-0.15	-0.13	-0.15	-0.31	-4.69	-5.38	-3.62	-3.91	-4.93
			h					$t(h)$		
Small	0.38	0.35	0.36	0.33	0.35	8.33	6.14	4.76	3.59	3.70
2	0.30	0.30	0.25	0.19	0.00	8.52	5.90	4.18	2.86	0.04
3	0.29	0.32	0.27	0.17	-0.10	7.36	5.49	4.80	2.64	-1.96
4	0.27	0.23	0.20	0.12	-0.12	4.83	3.87	3.57	2.22	-2.18
Big	0.09	-0.06	0.04	0.07	-0.12	3.25	-2.29	1.15	1.85	-2.10
			r					$t(r)$		
Small	0.16	0.31	0.30	0.21	-0.02	1.81	3.16	2.76	1.73	-0.16
2	0.10	0.25	0.30	0.35	-0.06	1.44	2.71	2.97	3.55	-0.69
3	0.08	0.25	0.37	0.34	-0.05	1.07	2.43	4.45	4.31	-0.76
4	0.07	0.24	0.30	0.27	-0.09	0.93	2.57	3.77	3.96	-1.04
Big	-0.00	0.14	0.21	0.05	-0.00	-0.04	3.90	4.23	0.93	-0.01
			c					$t(c)$		
Small	0.00	0.06	0.05	-0.04	0.02	0.07	0.97	0.58	-0.34	0.12
2	0.02	0.05	0.04	-0.01	-0.14	0.37	1.17	0.84	-0.17	-2.02
3	0.06	-0.03	0.07	0.01	-0.10	1.44	-0.59	1.29	0.16	-1.43
4	0.08	0.07	0.08	0.07	-0.03	1.25	1.17	1.35	0.94	-0.33
Big	-0.02	0.15	0.10	-0.03	-0.12	-0.45	3.30	2.05	-0.57	-1.77
			v					$t(v)$		
Small	-0.07	-0.03	0.08	0.22	0.52	-2.80	-0.83	2.17	4.48	10.33
2	-0.14	-0.11	-0.07	0.02	0.29	-6.88	-4.52	-2.79	0.73	6.28
3	-0.15	-0.10	-0.04	-0.01	0.26	-7.08	-3.63	-1.64	-0.47	8.13
4	-0.15	-0.10	-0.05	-0.00	0.27	-6.52	-4.14	-2.16	-0.16	8.80
Big	-0.12	-0.08	-0.03	0.03	0.27	-7.86	-5.17	-1.52	1.45	8.96

The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 Size-IV portfolios. The RHS variables are the excess market return $R_M - R_F$, the size factor SMB, the value factor HML, the profitability factor RMW, the investment factor CMA and IVFL, the return spread in univariate sort on conditional long-run idiosyncratic volatility. The sample period is July 1963 to December 2017.

Table 12: Summary Statistics for Tests of the FF-3, FF-5 and FF-5-plus-IVFL Models

Model Factors	GRS	$p(GRS)$	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|\bar{r}_i\|}$	$\frac{Aa_i^2}{A\bar{r}_i^2}$	$\frac{As^2(a_i)}{A\|\bar{r}_i\|}$	$A(R^2)$
MKT SMB HML	6.76	0.0	0.24	0.77	0.91	0.05	0.88
MKT SMB HML RMW CMA	5.55	0.0	0.13	0.43	0.37	0.12	0.9
MKT SMB HML RMW CMA IVFL	5.15	0.0	0.11	0.36	0.21	0.2	0.91

This table reports statistics summarizing how well the FF-3, FF-5 and FF-5-plus-IVFL models explain monthly excess returns on the 25 Size-IV portfolios. The table shows (1) the GRS statistics testing whether the expected values of all 25 intercept estimates are zero; (2) $p(GRS)$, the p-value for the GRS statistic; (3) the average absolute value of the intercepts, $A|a_i|$; (4) the average absolute value of the intercepts over the average absolute value of $\bar{r}_i$, which is the average excess returns on portfolio i minus the average market portfolio excess returns; (5) $Aa_i^2/A\bar{r}_i^2$, the average squared intercept over the average squared value of $\bar{r}_i$; (6) $As^2(a_i)/A|\bar{r}_i|$, the average of the estimates of the variances of the sampling errors of the estimated intercepts over $A\bar{r}_i^2$; and (7) $A(R^2)$, the average value of the regression R^2 corrected for degrees of freedom. The sample period is July 1963 to December 2017.

Appendix for the Short-Run and Long-Run Components of Idiosyncratic Volatility and Stock Returns

Yunting Liu [1]

A Empirical Analysis using the Smoothed Estimates of Short-Run and Long-Run Idiosyncratic Volatility

In this section, I present empirical results of using the smoothed estimates of short-run and long-run idiosyncratic volatility.

A.1 Portfolio Analysis with Smoothed Estimates

Table A1 reports the performance of portfolios sorted by the smoothed estimates of conditional long-run idiosyncratic volatility. Similar to the findings using filtered estimates, high long-run idiosyncratic volatility portfolios earn low returns. The quintile 5 portfolio with the highest smoothed long-run idiosyncratic volatility earns a return of -0.18% for the SL model, and as low as -0.56% for the PT model. The return spread between the quintile 1 and 5 portfolio for the PT model is -1.57%, with a t-statistic of -3.64. Compared to the spread of -1.19% using the SL model, the larger spread found in the PT model may arise because allowing for unit roots better captures the dynamics of idiosyncratic volatility. The potential existence of unit roots further supports the fact that there exists a highly persistent component in idiosyncratic volatility.

As for the conditional short-run component, Table A2 indicates that there is a significant positive relation between conditional short-run idiosyncratic volatility and expected stock returns. For the SL model, portfolios sorted by the filtered estimates $\hat{s}_t$ and the smoothed estimates $\tilde{s}_t$ both reveal positive relations between conditional short-run components and expected stock returns. The high-minus-low short-run volatility portfolio earns an average monthly return of 0.19% for the filtered estimates and 0.52% for the smoothed estimates. The statistical significance is also stronger for the smoothed estimates, with a statistical significance of 3.67 over 2.81 of the filtered estimates. This indicates that the model is relatively successful to capture the dynamics of idiosyncratic volatility

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and the relationship between idiosyncratic volatility and cross-sectional stock returns.

Table A1: Portfolios Sorted by the Smoothed Estimates of Conditional Long-Run Volatility

Panel A: Short- and Long-Run Volatility (SL) Model				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	1.00	3.59	47.1%	0.19
				[4.86]
2	0.99	4.81	32.3%	-0.02
				[-0.38]
3	0.95	6.40	13.6%	-0.14
				[-1.91]
4	0.53	8.43	5.6%	-0.74
				[-5.71]
5 (high)	-0.18	11.34	1.5%	-1.67
				[-6.53]
$5-1$	-1.19	9.87		-1.86
	[-2.70]			[-6.65]
Panel B: Permanent and Transitory Volatility (PT) Model				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	1.01	3.60	47.4%	0.19
				[4.94]
2	1.00	4.82	32.1%	-0.01
				[-0.16]
3	0.93	6.44	13.6%	-0.17
				[-2.17]
4	0.51	8.45	5.5%	-0.75
				[-5.66]
5 (high)	-0.56	11.19	1.4%	-2.03
				[-8.29]
$5-1$	-1.57	9.69		-2.23
	[-3.64]			[-8.27]

Portfolios are formed every month based on the smoothed estimates of long-run idiosyncratic volatility, $\tilde{l}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. The column $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column reports Jensen's alpha with respect to the Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics with up to one lag are reported in square brackets. The sample period is July 1963 to December 2017.

Table A2: Portfolios Sorted by Smoothed Estimates of Conditional Short-Run Volatility

Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.64	3.97	23.8%	-0.21
				[-3.97]
2	0.84	4.20	21.3%	-0.04
				[-1.03]
3	0.98	4.43	19.4%	0.07
				[1.71]
4	1.12	4.82	18.8%	0.17
				[3.54]
5 (high)	1.16	5.64	16.8%	0.09
				[0.95]
5-1	0.52	3.18		0.30
	[3.67]			[0.52]

Portfolios are formed every month based on conditional short-run idiosyncratic volatility of $\hat{s}_t$ or $\tilde{s}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. The column $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column reports Jensen's alpha with respect to the Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics with up to one lag are reported in square brackets. The sample period is July 1963 to December 2017.

A.2 Fama-MacBeth Regressions with Smoothed Estimates

In this section, I report Fama-MacBeth regression results using the smoothed estimates of conditional short-run and long-run idiosyncratic volatility. The results are very similar to those using the filtered estimates, except that using the smoothed estimates could lead to more significant regression coefficients.

Table A3: Relationship between Idiosyncratic Risk and Expected Returns: Cross-Sectional Evidence for the Short- and Long-Run Volatility (SL) Model

$\log v_t$	$\hat{s}_t$	$\hat{l}_t$	$\tilde{s}_t$	$\tilde{l}_t$	$Ret(-1)$	μ_t
-0.52						
[-5.66]						
	2.41	-0.50				
	[6.10]	[-4.01]				
			4.12	-0.75		
			[36.11]	[-5.74]		
-0.41					-4.78	
[-4.37]					[-10.62]	
	2.70	-0.48			-5.16	
	[6.81]	[-3.74]			[-11.43]	
			4.05	-0.79	-5.50	
			[35.69]	[-5.83]	[-12.26]	
	3.52	-0.34			-4.34	4.81
	[8.93]	[-2.66]			[-10.19]	[22.61]
			1.22	-1.06	-4.63	4.55
			[18.50]	[-8.21]	[-10.88]	[21.91]

The average coefficient is the time-series average of monthly regression coefficients from July 1963 to December 2017, and the t-statistic is the average coefficient divided by its time-series standard error. The t-statistic is reported in brackets.

Table A4: Relationship between Idiosyncratic Risk and Expected Returns: Cross-Sectional Evidence for the Permanent and Transitory Volatility (PT) Model

$\log v_t$	$\hat{l}_t$	$\tilde{l}_t$	$Ret(-1)$	μ_t
-0.52				
[-5.73]				
	-0.49			
	[-4.11]			
		-0.49		
		[-3.56]		
-0.42			-4.79	
[-4.55]			[-10.59]	
	-0.48		-5.07	
	[-3.90]		[-11.20]	
		-0.55	-5.65	
		[-3.83]	[-12.47]	
	-0.26			4.88
	[-2.12]			[23.34]
		-1.05		5.11
		[-8.18]		[26.06]
	-0.26		-4.19	4.75
	[-2.08]		[-9.72]	[22.90]
		-1.08	-4.51	4.96
		[-8.16]	[-10.42]	[25.69]

The average coefficient is the time-series average of monthly regression coefficients from July 1963 to December 2017, and the t-statistics is the average coefficient divided by its time-series standard error. The t-statistic is reported in brackets.

B Further Investigation of the Long-Run Component of Idiosyncratic Volatility

In this section, I further investigate the long-run component of idiosyncratic volatility by separating the constant term ϕ_i from the long-run volatility defined previously. Recall that the short-run and long-run idiosyncratic volatility model is defined as

$$\begin{aligned} \textbf{Idiosyncratic Volatility} &: \log v_t^i = s_t^i + l_t^i \\ \textbf{Short-Run Component} &: s_{t+1}^i = \rho_s^i s_t^i + \sigma_s^i \epsilon_{s,t}^i \\ \textbf{Long-Run Component} &: l_{t+1}^i = \phi_i + \rho_l^i l_t^i + \sigma_l^i \epsilon_{l,t}^i \end{aligned} \tag{1}$$

Let $\hat{\phi}_t \equiv \mathbb{E}(\phi|y_1, y_2, y_{t-1})$ denotes the expectation of the constant term based on information available at time $t-1$. The expectation of the long-run component is already defined as $\hat{l}_t \equiv \mathbb{E}_{t-1}(l_t|y_1, y_2, ...y_{t-1})$. Thus the term $\hat{l}_t - \hat{\phi}_t$ captures the expectation of the long-run component zeroing out the constant term.

Table B1 reports the performance of portfolios sorted by the long-run component zeroing out the constant, $\hat{l}_t - \hat{\phi}_t$. The portfolio with the highest idiosyncratic volatility (quintile 5) has an average return of 0.79% per month. The difference in returns between the highest and lowest idiosyncratic volatility portfolio is -0.20% per month, with a robust t-statistic of -1.31. Portfolio analysis results for the expected constant term $\hat{\phi}_t$ are reported in Table B2. The portfolio with the highest $\hat{\phi}_t$ earns 0.50% per month. The return spread between portfolios the highest and lowest ϕ_t is -0.38% with a robust t-statistic of -1.19.

Thus, both parts of the long-run idiosyncratic volatility are negatively related to cross-sectional stock returns. However, both the magnitude of return spreads and their statistical significance are reduced from separating the long-run component into the constant and non-constant part. By comparison, the return spread between highest and lowest expected long-run idiosyncratic volatility -0.73% per month with a robust t-statistic of -2.10. This suggests that the long-run component with the constant term incorporates more information and is more predictive for cross-sectional stock returns.

Table B1: Portfolios Sorted by the Idiosyncratic Volatility $\hat{l}_t - \hat{\phi}_t$

Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha	Average $\hat{l}_t - \hat{\phi}_t$
1 (low)	0.99	4.67	29.8%	0.17	-0.30
				[2.90]	
2	0.96	4.78	23.2%	0.12	-0.11
				[2.23]	
3	0.89	4.57	18.3%	-0.00	-0.00
				[-0.05]	
4	0.85	4.58	15.7%	-0.11	0.10
				[-2.12]	
5 (high)	0.79	5.30	13.0%	-0.33	0.35
				[-4.01]	
5-1	-0.20	3.37		-0.5	
	[-1.31]			[-4.34]	

Portfolios are formed every month based on the idiosyncratic volatility, $\hat{l}_t - \hat{\phi}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. The column $5 - 1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column reports Jensen's alpha with respect to the Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics with up to one lag are reported in square brackets. Average $\hat{l}_t - \hat{\phi}_t$ is computed as the simple average of the long-run component zeroing out the constant. The sample period is July 1963 to December 2017.

Table B2: Portfolios Sorted by the Idiosyncratic Volatility $\hat{\phi}_t$

Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha	Average $\hat{\phi}_t$
1 (low)	0.88	3.80	44.3%	0.02	1.50
				[0.56]	
2	0.99	4.83	31.2%	0.03	1.93
				[0.69]	
3	1.04	5.98	15.8%	0.05	2.22
				[0.77]	
4	0.99	7.63	6.8%	-0.07	2.51
				[-0.64]	
5 (high)	0.50	9.40	1.9%	-0.69	2.90
				[-4.43]	
5-1	-0.38	7.61		-0.71	
	[-1.19]			[-4.1]	

Portfolios are formed every month based on the idiosyncratic volatility, $\hat{\phi}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std.Dev. are measured in monthly percentage terms and apply to total, not excess returns. The column $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column report Jensen's alpha with respect to the Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics with up to one lag are reported in square brackets. Average $\hat{\phi}_t$ is computed as the simple average of the expected constant term ϕ_t in the long-run component. The sample period is July 1963 to December 2017.

Table B3: Portfolios Sorted by the Filtered Estimates of Expected Long-Run Volatility $\hat{l}_t$

Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha	Average $\hat{l}_t$
1 (low)	0.92	3.68	47.2%	0.09	1.45
				[2.50]	
2	1.00	4.77	32.0%	0.00	1.90
				[0.06]	
3	1.05	6.21	13.7%	-0.01	2.21
				[-0.1]	
4	0.86	7.98	5.6%	-0.31	2.52
				[-2.73]	
5 (high)	0.18	9.88	1.5%	-1.15	3.00
				[-6.26]	
5-1	-0.73	8.26		-1.25	
	[-2.10]			[-6.06]	

Portfolios are formed every month based on the idiosyncratic volatility, $\hat{l}_t$. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. The column $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column reports Jensen's alpha with respect to the Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics with up to one lag are reported in square brackets. Average $\hat{l}_t$ is computed as the simple average of the expected long-run component. The sample period is July 1963 to December 2017.

B.1 Transition Probabilities Across Quintile Portfolios

Table B4 below reports the transition probabilities across quintile portfolios sorted by the filtered long-run or short-run idiosyncratic volatility. For the long-run volatility, the probability of staying in the same portfolio at month $t-1$ and t ranges 75.93% to 90.30% as reported in Panel A. High probabilities along the diagonal shows that conditional long-run volatility is relatively persistent.

Panel B of Table B4 presents the results for the filtered short-run volatility. The probability of staying in the same quintile portfolio ranges from 22.81% to 52.11%. These relatively low values indicate that the short-run volatility is short-lived and die off quickly.

Table B4: Portfolio Transition Probabilities

Panel A: The Filtered Long-Run Volatility						
				Month t		
		1 (low)	2	3	4	5 (high)
Month $t-1$	1 (low)	90.30	9.40	0.26	0.03	0.01
	2	9.34	78.82	11.41	0.39	0.04
	3	0.21	11.43	75.93	12.08	0.35
	4	0.05	0.29	12.01	77.42	10.23
	5 (high)	0.02	0.04	0.24	9.97	89.74

Panel B: The Filtered Short-Run Volatility						
				Month t		
		1 (low)	2	3	4	5 (high)
Month $t-1$	1 (low)	31.10	15.633	6.22	15.50	31.55
	2	15.87	26.12	17.48	24.96	15.56
	3	6.37	17.55	52.11	17.76	6.22
	4	15.54	25.15	17.86	26.12	15.33
	5 (high)	30.97	15.70	6.24	15.45	22.81

This table reports the transition probabilities across portfolios sorted by the filtered long-run or short-run idiosyncratic volatility. The numbers in the table are expressed in percentage terms.

C Time-Series Plots of Idiosyncratic Volatilities

In this section, I plot a few time-series of the filtered short-run and long-run components of idiosyncratic volatility. The randomly selected firms are Apple Inc, Nordstrom Inc, and Gilead Science Inc. [2] As we can see from these graphs, the short-run component mostly fluctuates around mean, zero. And it displays little persistence. On the contrary, variations in the long-run component are relatively long-lived. The level of the long-run component could have persistent variations over time and across firms. I also plot the filtered constant term ϕ_t and the filtered long-run component zeroing out the constant term, $\hat{l}_t - \hat{\phi}_t$. We can see that these two parts of the long-run idiosyncratic volatility could have independent variations, and are not necessarily perfectly correlated.

[2] Their CRSP identifier PERMNO are 14593, 57817, and 77274 respectively.

Figure C1: Time-Series Plots of Different Measures of Idiosyncratic Volatility for Apple

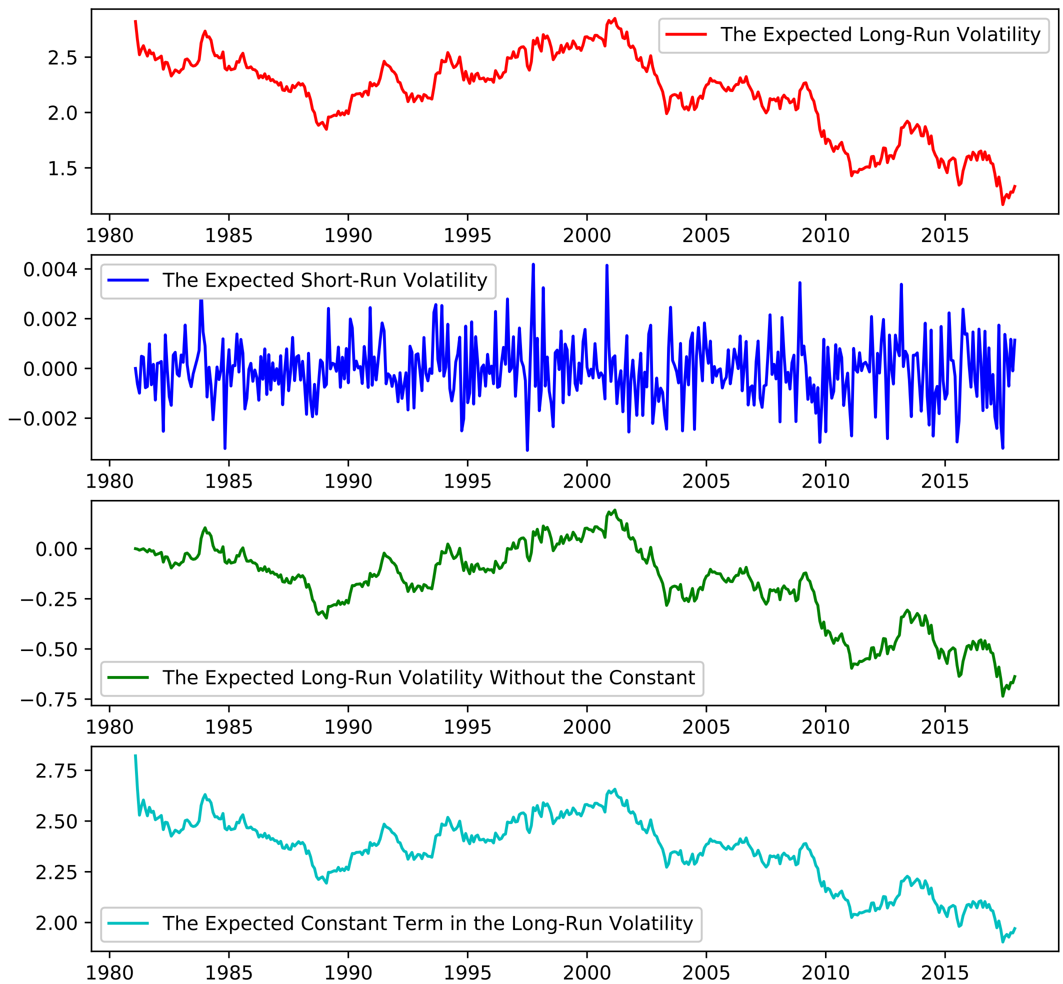


Figure C2: Time-Series Plots of Different Measures of Idiosyncratic Volatility for Nordstrom

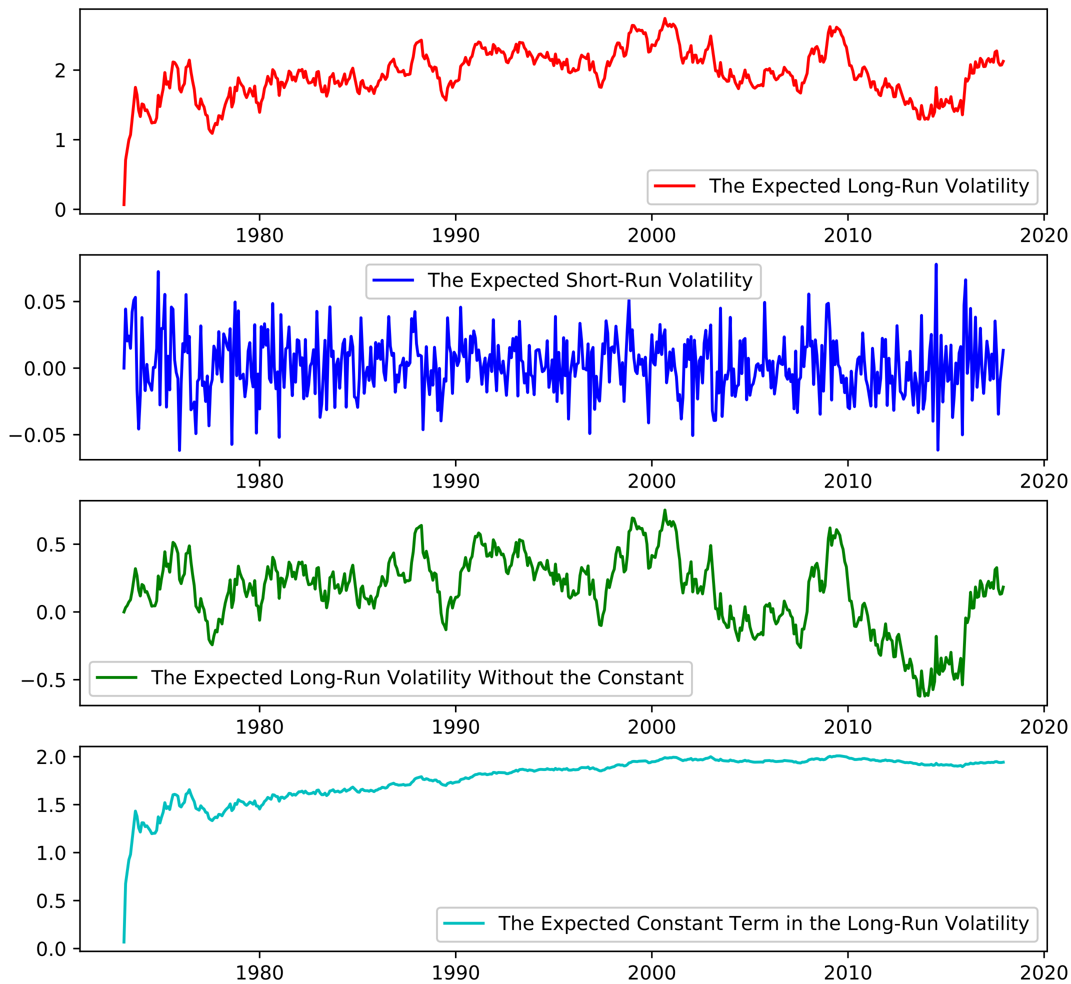
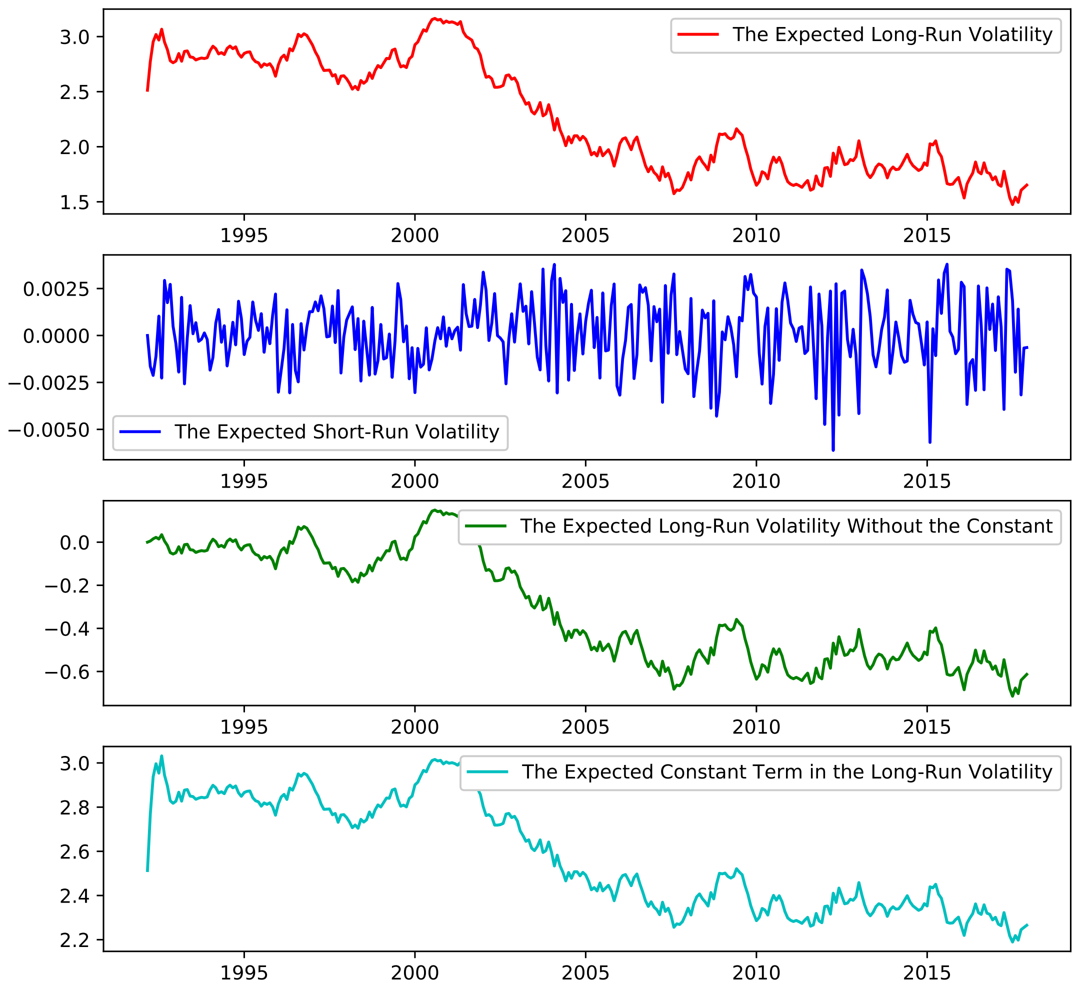


Figure C3: Time-Series Plots of Different Measures of Idiosyncratic Volatility for Sprint



D Estimating the Short-Run and Long-Run Idiosyncratic Volatility Model with Measurement Errors

In this section, I estimate the short-run and long-run volatility model with measurement errors in realized idiosyncratic volatility. The short-run and long-run idiosyncratic volatility model in the paper is defined as

$$\begin{aligned}\textbf{Idiosyncratic Volatility}: & \log v_t^i = s_t^i + l_t^i \\ \textbf{Short-Run Component}: & s_{t+1}^i = \rho_s^i s_t^i + \sigma_s^i \epsilon_{s,t}^i \\ \textbf{Long-Run Component}: & l_{t+1}^i = \phi_i + \rho_l^i l_t^i + \sigma_l^i \epsilon_{l,t}^i\end{aligned} \tag{2}$$

In the paper, I report results treating the realized idiosyncratic volatility IV_t as measuring the true latent volatility v_t^i without errors. In this section, I relax this assumption and introduce measurement errors in the following way. I model the log of realized volatility as the log of latent volatility v_t plus a identically and independently distributed measurement error $\epsilon_{u,t}$

$$\log IV_t^i = \log v_t^i + \epsilon_u = s_t^i + l_t^i + \epsilon_{u,t} \tag{3}$$

The standard error of $\epsilon_{u,t}$ is denoted by σ_u, which is also to be estimated. The specification of Equation 3 is still in a state-space from and can be estimated using a Kalman filter. The following Table D1 reports parameter estimates of the model with measurement errors. As we can see, estimates of the persistence parameters are not significantly affected by the introduction of measurement errors. With measurement errors, the median of persistence parameters ρ_l and ρ_s are 0.04 and 0.94. By comparison, they are -0.003 and 0.94 without measurement errors. The introduction of measurement errors does reduce estimates of the standard deviation of shocks to the short-run component. The median of σ_s is reduced from 0.31 to 0.22. The median of the standard deviation of shocks to the long-run component, i.e., σ_l slightly decreases from 0.15 to 0.14.

However, the median of the standard deviations of measurement errors is 0.0001. This suggests that for a fair amount of firms the data is not rich enough to efficiently separate the measurement error from the short-run component. This problem could be severe if the short-run component is close to being white noise. It would then be hard to distinguish between the short-run component and the measurement error.

Table D1: Parameter Estimates of Idiosyncratic Volatility Model

Panel A: The Short and Long Run Volatility (SL) Model with Measurement Errors					
Variables	ρ_s	ρ_l	σ_s	σ_l	σ_u
Mean	-0.004	0.77	0.21	0.19	0.12
Median	0.04	0.94	0.22	0.14	0.0001

Panel B: The Short and Long Run Volatility (SL) Model without Measurement Errors				
Variables	ρ_s	ρ_l	σ_s	σ_l
Mean	-0.07	0.79	0.29	0.20
Median	-0.003	0.94	0.31	0.15

This table summarizes the properties of parameter estimates for the short-run and long-run idiosyncratic volatility processes. I first compute parameter estimates for each stock and then construct the mean and median statistics across all stocks. The sample period is July 1963 to December 2017.

The following table D2 reports portfolio analysis results using the filtered estimates of the short-run and long-run components. It can be seen that most results are very similar to those without measurement errors. The spread between the quintile 1 and quintile 5 portfolio is about 0.18% for the short-run component and -0.65% for the long-run model. These slightly smaller return spreads compared to those without measurement errors may be caused by reduced precision in identifying the short-run and long-run components.

Table D2: Portfolios Sorted by the Filtered Estimates of Conditional Idiosyncratic Volatility with Measurement Errors

Panel A: Filtered Estimates of the Long-Run Idiosyncratic Volatility				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.94	3.72	46.2%	0.11
				[2.93]
2	1.00	4.82	32.5%	0.01
				[0.31]
3	1.02	6.14	14.2%	-0.03
				[-0.45]
4	0.84	7.80	5.7%	-0.33
				[-2.98]
5 (high)	0.29	9.67	1.3%	-1.07
				[-6.32]
5-1	-0.65	8.09		-1.18
	[-1.87]			[-6.24]

Panel B: Filtered Estimates of the Short-Run Idiosyncratic Volatility				
Rank	Mean	Std. Dev.	% Mkt Share	FF-3 alpha
1 (low)	0.84	4.54	14.7%	-0.06
				[-1.17]
2	0.94	4.48	22.4%	0.05
				[1.20]
3	0.88	4.59	25.5%	-0.04
				[-1.29]
4	0.95	4.44	23.2%	0.06
				[1.51]
5 (high)	0.98	4.53	14.2%	0.04
				[0.78]
5-1	0.18	1.90		0.10
	[1.77]			[1.36]

Portfolios are formed every month based on the filtered estimates of the conditional short-run or long-run idiosyncratic volatility. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, returns. The column $5-1$ refers to the difference in monthly returns between portfolio 5 and portfolio 1. The alpha column reports Jensen's alpha with respect to the Fama-French (1993) three-factor model. Robust Newey-West (1987) t-statistics with up to one lag are reported in square brackets. The sample period is July 1963 to December 2017.

E Monte Carlo Simulations

Because realized idiosyncratic volatility is strongly time-varying and has an average first-order auto-correlation of about 0.39, Fu (2009) proposes that exponential generalized autoregressive conditional heteroscedasicity (EGARCH) model could be used to capture the short-run variation in conditional idiosyncratic volatility. Subsequent papers by Guo et al. (2014) and Fink et al. (2012) argue that the EGARCH estimates of the conditional idiosyncratic volatility by Fu (2009) could be subject to substantial look-ahead biases. The positive relation between the conditional idiosyncratic volatility predicted by EGARCH model and cross-sectional stock returns found by Fu (2009) is not robust. This section also investigates whether the EGARCH model is useful to capture variations in the short-run component of idiosyncratic volatility. I conduct Monte-Carlo simulations to study this question. Simulation results find that the EGARCH predicted conditional volatility is positively correlated with the long-run component as opposed to the short-run component. And the conditional idiosyncratic volatility predicted by EGARCH model is negatively related to cross-sectional stock returns. This finding highlights that it is useful to model the short-run and long-run components jointly to capture the dynamics idiosyncratic volatility.

E.1 A Discrete Time Model for Monte Carlo Simulations

Consider a simple discrete time model in which the daily returns of an individual stock are characterized as

$$r^i_{t,d} = \sigma^i_t \epsilon^i_{t,d} + \gamma_s s^i_t + \gamma_l l^i_t \tag{4}$$

where for day d in month t, $r^i_{t,d}$ is stock i's excess return. The residuals $\eta^i_{t,d} \equiv \sigma^i_t \epsilon^i_{t,d}$ is the idiosyncratic risk for month t whereas σ^i_t is the standard deviation of the residual. The following definition is consistent with the notation in Section 2 of the paper. I define the idiosyncratic volatility of stock returns for firm i in month t as v^i_t

$$v^i_t = \sigma^i_t \sqrt{N_m}$$

where N_m is the number of trading days in month t for firm i. The volatility dynamics for v_t^i follows the specification in the previous section as

$$\begin{aligned}\textbf{Idiosyncratic Volatility}: & \log v_t^i = s_t^i + l_t^i \\ \textbf{Short-run Component}: & s_{t+1}^i = \rho_s^i s_t^i + \sigma_s^i \epsilon_{s,t}^i \\ \textbf{Long-run component}: & l_{t+1}^i = \phi_i + \rho_l^i l_t^i + \sigma_l^i \epsilon_{l,t}^i\end{aligned}$$

The log-volatility is the sum of two components, s_t and l_t. The parameter γ_s and γ_l capture the relationship between short-run and long-run component of idiosyncratic volatility and cross-section of stock returns. For example, when $\gamma_s = 0$, there is no relationship between conditional short-run idiosyncratic volatility and expected stock returns. It is worth emphasizing this is a very reduced form way of capturing the cross-sectional relationship between idiosyncratic volatility and stock returns. I remain silent on whether the cross-sectional relation is related to risk or not.

Without loss of generality, I assume the risk-free rate to be zero. For each stock, the monthly excess return then takes the following form

$$R_t^i = \prod_{d=1}^{N_m} (1 + r_{t,d}^i) - 1$$

In the simulation, these parameters are set as follows based on empirical estimates using the CRSP data. I set γ_s to be 2.41 and γ_l to be -0.50, which are the average regression coefficients in the Fama-Macbeth regressions using filtered estimates. The parameter of the process is set as as $\rho_l = 0.94$ which is the median persistence parameter of the long-run component. The persistence of short-run component is set to be $\rho_s = -0.003$, the median persistence parameter of the short-run component. And σ_s and σ_l are the volatility of shocks to the short-run and long-run components, which can be set to 0.31 and 0.15 respectively based on the empirical estimates. The constant ϕ_i is set to be uniformly distributed with a mean of 2.43 with a standard deviation of 0.62. The numbers correspond to the empirical estimates using CRSP data. In simulated data, each stock has 230 months of daily stock returns observations, which is the median number of daily stock observations for CRSP common stocks. For illustration, I investigate cross-sectional implications using 1000 stocks; more stocks do not affect the results in any statistically significant manner.

In the simulated data, the autocorrelations of realized idiosyncratic volatility decays quickly over short periods but slower over longer periods. The first order autocorrelation of realized volatility is 0.42, second order to be 0.39, third order to be 0.35 and the autocorrelation of 12 months lag is 0.16. These autocorrelations approximately match the empirical estimates reported in the Panel B of Table 1 in the paper.

To evaluate the quantitative relationship between the conditional idiosyncratic volatility predicted by the EGARCH model and cross-sectional stock returns, I consider Fama–MacBeth regressions by regressing stock returns on conditional EGARCH volatility obtained using the full-sample of simulated data. The EGARCH(p,q) model is modeled as [3]

$$R_t^i = \alpha_t^i + \epsilon_t^i \tag{5}$$

where α_t^i is the intercept term, ϵ_t^i is assumed to have a serially independent normal distribution $\epsilon_t^i \sim N(0, {\sigma_t^i}^2)$. The conditional variance ${\sigma_t^i}^2$ follows

$$\log {\sigma_t^i}^2 = a_i + \sum_{l=1}^{p} b_{i,l} \log {\sigma_{t-l}^i}^2 + \sum_{k=1}^{q} c_{i,k} \left\{ \theta \left(\frac{\epsilon_{t-k}^i}{\sigma_{t-k}^i} \right) + \kappa \left[\left| \frac{\epsilon_{t-k}^i}{\sigma_{t-k}^i} \right| - \left(\frac{2}{\pi} \right)^{1/2} \right] \right\} \tag{6}$$

Estimating EGARCH model produces an average coefficient of b_1 as 0.91 and the coefficient of c_1 is 0.61. Therefore, the persistence parameter of idiosyncratic volatility b_1 in EGARCH model is largely influenced by the long-run dynamics of idiosyncratic volatility. And the average regression coefficient of stock returns on the conditional idiosyncratic volatility obtained by EGARCH model is -0.65, with a t-statistic of -3.59. This indicates that the conditional volatility predicted by the EGARCH model largely captures the conditional volatility of the long-run component instead of the short-run component. This is intuitively plausible because the conditional volatility is a positive function of lagged volatility, which largely consists of the persistent long-run component. Hence, we would expect a negative relationship between the conditional volatility estimated by the EGARCH model and expected stock returns. These simulation results demonstrate that modeling the short-run and long-run components of idiosyncratic volatility jointly is crucial to capture the information of conditional volatility over different horizons.

F Additional Forecasting Regressions

In this section, I run additional forecasting regressions to investigate whether the conditional long-run idiosyncratic volatility is useful to forecast market returns[4]. As explained by Campbell (1992), the Intertemporal Asset Pricing Model (e.g., Merton 1973) suggests that variables that proxy for the aggregate investment opportunities should also forecast stock-market returns. If the

[3]I report the EGARCH(1,1) estimates while EGARCH with different lags produce similar results. I also consider specifications of estimating the time t EGARCH volatility recursively using the information available up to time $t-1$. Since stock returns are not skewed in my simulation, the look-ahead bias documented by Guo et al. (2014) doesn't exist. Hence, similar results are produced using full sample and information up to $t-1$.

[4]Thank one referee for suggesting this point.

conditional long-run idiosyncratic volatility is a proxy for investment opportunities, it may also forecast market returns. Following Guo and Savickas (2010), I use the returns on stocks with high long-run idiosyncratic volatility in the previous twelve quarters to forecast market returns in the following four quarters. The following table reports that the predictive power is strongest for the quintile portfolio with the highest volatility. The t-value is -4.65, and the R^2 is 0.11. By comparison, these statistics are -2.0 and 0.02 for the quintile portfolio with the lowest conditional long-run volatility. These regression results lend support to the story that conditional long-run idiosyncratic volatility is positively related to future investment opportunities.

Table F1: Forecasting Regressions Results

Rank	Coefficients	t-value	R^2
1 (low)	-0.30	-2.00	0.02
2	-0.46	-2.72	0.03
3	-0.50	-3.88	0.07
4	-0.38	-3.52	0.06
5 (high)	-0.37	-4.65	0.11

This table reports forecasting regressions statistics of regressing market returns on returns of portfolios sorted by the conditional long-run idiosyncratic volatility. The sample period is July 1963 to December 2017. The coefficients column reports regression coefficients of idiosyncratic volatility portfolio returns, and the t-value column reports robust Newey-West (1987) t-statistics with up to one lag. The R^2 column shows regression R^2s.

G The Model

In this section, I construct a model similar to the models of Berk et al. (1999) and Carlson et al. (2004) to shed light on the negative relation between idiosyncratic volatility and stock returns. In particular, the model may be useful to explain why the long-run idiosyncratic volatility could be negatively related to cross-sectional stock returns.

G.1 The environment

Assume that each firm i, $i \in \{1, ...I\}$ produces a single commodity that can be sold at time-t in the product market with productivity $P_{i,t}$,

$$P_{i,t} = X_{i,t} Z_t \tag{7}$$

where $X_{i,t}$ is the idiosyncratic productivity component and Z_t is the aggregate productivity component with dynamics

$$dZ_t/Z_t = \mu dt + \sigma dB_t \tag{8}$$

$$dX_{k,t}/X_{k,t} = \sigma_i dB_{i,t} \tag{9}$$

The parameter μ denotes the growth rate of aggregate productivity, σ the volatility of aggregate productivity, σ_i the volatility of idiosyncratic productivity, and $dB_{i,t}$ and dB_t are the increments of two independent Brownian motions, The innovation to idiosyncratic productivity $dB_{i,k}$ is independent across firms. Let $p_{i,t}$ denotes the log of productivity for firm i, the law of motion for $p_{i,t}$ would therefore be

$$dp_{i,t} = \mu_p dt + \sigma dB_t + \sigma_i dB_{i,t} \tag{10}$$

where $\mu_p = \mu - \frac{1}{2}\sigma^2 - \frac{1}{2}\sigma_i^2$

Following several papers investigating the cross-section of equity returns (Berk et al. (1999), Carlson et al. (2004), Zhang (2005))), I assume a stochastic discount factor with the following process:

$$\frac{d\pi_t}{\pi_t} = -rdt - \theta dB_t \tag{11}$$

where θ is the constant market price of risk, and r is the risk-free rate. Exposure to aggregate productivity shocks is priced while exposure to idiosyncratic productivity shocks is not.

We can carry out the valuation of firms under the risk-neutral measure $\mathbb{Q}$. Working under $\mathbb{Q}$ measure, the dynamics of the aggregate component of the productivity is

$$dZ_t/Z_t = \hat{\mu}dt + \sigma dB_t \tag{12}$$

where the risk neutral drift $\hat{\mu} = \mu - \sigma\theta$.

G.2 Assets in Place

An asset in place is a project that generates payoffs $Y_{i,t}$ with scale S

$$Y_{i,t} = P_{i,t}S \tag{13}$$

where $P_{i,t}$ is the productivity of firm i. Let the production cost be C per unit of output, so the profit at time t is $P_{i,t} - c$. Therefore, the valuation of an asset in place is

$$\begin{aligned} A(p_{i,t}) &= \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(\omega-t)}(P_{i,t} - C)Sd\omega\right] \\ &= \frac{\exp(p_{i,t})}{r-\hat{\mu}}S - \frac{C}{r}S \end{aligned} \tag{14}$$

Because idiosyncratic risk is not priced, the value of assets in place is independent of the idiosyncratic productivity volatility.

The dynamics for $A(p_{i,t})$ therefore follows

$$dA = SdP_{i,t} = \frac{S}{r-\hat{\mu}}P_{i,t}\left(\mu dt + \sigma dB_t + \sigma_{i,t}dB_{i,t}\right) \tag{15}$$

G.3 Growth Options

A growth option is the right to obtain the above project by making an irreversible investment of K units of the goods. The firm is endowed with an opportunity to obtain such project and can choose when it is optimal to do so. The optimal policy is to exercise the option if the log productivity $p_{i,t}$ is above the threshold p_i^*. Let $G(p_{i,t}, p_i^*)$ denotes the value of the growth option and τ denotes the moment the option is exercised. The value of the growth option is determined by the current level of productivity and the threshold to exercise the option P_i^*. When the current productivity $p_{i,t}$ is above p_i^*, the growth is exercised immediately.

Prior to exercise, $t < \tau$, the expected present-value of the project's payoff gives the value of the option

$$G(p_{i,t}, p_i^*) = \mathbb{E}_t^Q\left[e^{-r(\tau-t)}\Big(A(p_i^*) - K\Big)\right];\ t \le \tau \tag{16}$$

I prove the following proposition in Section H.

Proposition 1. *The value of the growth option is given by*

$$G(p_{i,t}) = \begin{cases} A(p_i^*) - K; & p_{i,t} \ge p_i^* \\ \left(\frac{P_{i,t}}{P_i^*}\right)^{l_2}\Big(A(p_i^*) - K\Big); & p_{i,t} < p_i^* \end{cases} \tag{17}$$

where

$$p_i^* = \frac{l_2[C * S + K(r - \hat{\mu}_{p,i})]}{(l_2 - 1)S}; \quad l_2 = \frac{-\hat{\mu}_{p,i} + \sqrt{\hat{\mu}_{p,i}^2 + 2r\sigma_{p,i}^2}}{\sigma_{p,i}^2}$$
$$\hat{\mu}_{p,i} = \hat{\mu} - \frac{1}{2}(\sigma^2 + \sigma_i^2); \quad \sigma_{p,i} = \sqrt{\sigma^2 + \sigma_i^2}$$

Prior to exercise, the value of the growth option depends on the level of idiosyncratic volatility. Because $\partial l_2/\partial\sigma_i < 0$, ceteris paribus, the value of the growth option is increasing in the volatility of idiosyncratic productivity shocks. This is an important valuation property that growth options have due to Jensen's inequality. Hence, the option is worth more if the idiosyncratic volatility of productivity is higher.

G.4 Expected Return

Having discussed the valuation, I now turn to the model-implied returns. The return on assets in place is independent of production scale because the the value of assets in place displays constant return to scale with respect to production scale. The return of assets $R_{A,t}$ can derived from (14) as

$$R_{A,t} = \frac{dA_t + S(P_t - C)dt}{A_t} \tag{18}$$
$$= (r - \hat{\mu})dt + L(p_t)\Big(\mu dt + \sigma dB_t + \sigma_{i,t}dB_{i,t}\Big) \tag{19}$$

where $L(p_t) = \frac{P}{r-\hat{\mu}}/(\frac{P}{r-\hat{\mu}} - \frac{c}{r})$. The return on assets in place is exposed to both aggregate idiosyncratic productivity shocks and idiosyncratic productivity shocks and risk exposures are amplified by a factor $L(p_t)$, which captures the operating leverage.

The return on growth options is given by

$$R_{G,t} = \frac{dG_t}{G_t} = l_2(u_p dt + \sigma dB_t + \sigma_i dB_{i,t}) \tag{20}$$

The exposure to aggregate and idiosyncratic productivity shocks is amplified by the factor l_2. In particular, the factor l_2 is decreasing in idiosyncratic volatility σ_i. This implies that the higher is the idiosyncratic volatility of productivity shocks, the lowers is the expected return on the option itself.

The intuition is as follows. When idiosyncratic volatility increases, the value of growth options could rise due to convexity. In the meantime, growth options' sensitivity to systematic risk factors could decrease because the relative magnitude of such options' value that is related to systematic risk falls. This channel drives down the expected return when idiosyncratic volatility is higher. Therefore, long-run idiosyncratic volatility serves as a proxy for exposure to systematic risk factors.

G.5 Why Are Long-Run Idiosyncratic Volatility Related to Cross-Sectional Stock Returns?

This paper has demonstrated empirical results that there is a negative relationship between the conditional long-run idiosyncratic volatility and cross-sectional stock returns. Such relation may be interpreted through the theoretical model in this Section G. In this real-option-based model, ceteris paribus, high idiosyncratic volatility increases the value of growth options, which lowers the exposure of firm equity to systematic risk factors. Therefore, firms with higher idiosyncratic volatility would have lower average returns. This implication is delivered from a comparative static sense and therefore the difference in volatility is assumed to be permanent. However, if such difference in volatility is relatively persistent over time, the comparative static result could give a close approximation. Thus, the persistent long-run idiosyncratic volatility could be negatively related to cross-sectional stock returns. And the long-run idiosyncratic volatility is proxying for exposures to systematic risk factors.

H Proof of Proposition 1

Using $p_t = lnP_t$ as the state variable, we have (using Ito's Lemma and Girsanov's Theorem) under $\mathbb{Q}$,

$$dp = \hat{\mu_p} dt + \sigma_p d\hat{B}_t \tag{21}$$

where $\hat{\mu}_{p,i} = \hat{\mu} - \frac{1}{2}(\sigma^2 + \sigma_i^2)$; $\sigma_{p,i} = \sqrt{\sigma^2 + \sigma_i^2}$, $d\hat{B}_t = \frac{\sigma}{\sigma_p} dB_t + \frac{\sigma_i}{\sigma_p} dB_{i,t}$,

Prior to exercise, the associated Hamilton-Jacobi-Bellman (HJB) equation is

$$rG(p, p_i^*) = \hat{\mu}_{p,i} G_p(p, p_i^*) + \frac{1}{2}\sigma_{p,i}^2 G_{pp}(p, p_i^*), \quad \text{all } p < p_i^* \tag{22}$$

This is a second order linear ordinary equation with constant coefficients. Hence any solution has the form

$$G(p_{i,t}) = c_1 e^{l_1 p_{i,t}} + c_2 e^{l_2 p_{i,t}} \tag{23}$$

where c_1, c_2 are constants that must be determined from boundary conditions. And

$$l_1 = \frac{-\hat{\mu}_{p,i} - J}{\sigma_{p,i}^2} \leq 0, \qquad l_2 = \frac{-\hat{\mu}_{p,i} + J}{\sigma_{p,i}^2} \geq 0, \qquad J \equiv \left(\hat{\mu}_{p,i}^2 + 2r\sigma_{p,i}^2\right)^{1/2} \geq \mu \tag{24}$$

To ensure that the growth option's value is finite when $p_{i,t}$ tends to $-\infty$, $c_1 = 0$.

Value matching at p_i^* yields

$$c_2 = \frac{A(P_i^*) - K}{P_i^{*l_2}}$$

Therefore, the value of the growth option is given by

$$G(p_{i,t}) = \begin{cases} A(P_i^*) - K; & p_{i,t} \geq p_i^* \\ \left(\frac{P_{i,t}}{P_i^*}\right)^{l_2} \Big(A(p_i^*) - K\Big); & p_{i,t} < p_i^* \end{cases} \tag{25}$$

Smooth pasting condition $\partial G / \partial P_i^*$ continuous at P_i^* yields

$$\frac{A'(p_i^*)}{A(p_i^*) - K} = l_2 \tag{26}$$

Hence, the threshold

$$p_i^* = \frac{l_2[C * S + K(r - \hat{\mu}_{p,i})]}{(l_2 - 1)S}$$

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